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THE

FIRST PRINCIPLES

OF

NATURAL PHILOSOPHY.

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NATURAL PHILOSOPHY.

BY

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PREFACE.

In offering this little Work to the public, it will be proper to commence by stating its object and extent.

Its object is to explain clearly, accurately, and in small compass, the fundamental principles of those parts of Natural Philosophy of which it treats. My own experience in tuition has convinced me of the desirableness of such a book, and has, perhaps, also qualified me, in some measure, for the task I have undertaken, of supplying what appeared to me to be a need.

It is divided into five sections, corresponding with those five branches of Natural Philosophy, to the study of which it is designed to be an Introduction.

The first two sections are devoted to Mechanics, divided into Statics and Dynamics. In Statics I have commenced by proving the great principle of Equilibrium, known under the name of the

Parallelogram of Forces; have applied it to showing how to find the conditions of equilibrium on the principal mechanical powers; and have explained the meaning and properties of the Centre of Gravity. In Dynamics, I have endeavoured to place in a clear light the Laws of Motion, and their consequences, particularly as instanced in the Fall of bodies; and in this subject have thought it desirable to illustrate the principles by a few examples, which are not more in number than is sufficient for that purpose. The third section is on Hydrostatics and Hydrodynamics; its object is to show the principal points in which the equilibrium and motion of fluid bodies differ from those of solid bodies; while the fourth section, on Pneumatics, explains the peculiar properties of such fluids as are elastic, describing also some ordinary instruments, the action of which depends upon the pressure of the atmosphere. Lastly, in the fifth section, on Optics, I have explained how the image of an object is formed, either by the eye itself, which acts precisely as an optical instrument does, or by a lens or mirror, to be afterwards viewed by the eye, giving the laws of reflexion and refraction, and taking occasion to describe the principle of the microscope and the telescope.

I have given demonstrations of all the principles laid down, so far as perfectly satisfactory ones are known; but have not employed any mathematical reasoning out of the reach of any one who has read the early books of Euclid and ordinary elementary algebra.

Greenwich,
October 1863.

W. T. L.

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THE FIRST PRINCIPLES

OF

NATURAL PHILOSOPHY.

SECTION I.

MECHANICS.—STATICS.

1. MECHANICS is the science which investigates and measures the effects of Force. Whatever produces or tends to produce motion in a body is called a Force. If motion actually is produced, the investigation is called Dynamics; if one force is so counteracted by another force that no motion ensues, it takes the name of Statics. Dynamics therefore may be said to include Statics; but as the investigation of balanced forces is much simpler than that of those which produce motion, it is usual to treat of Statics separately, and before Dynamics.

2. We must premise that, in the measurement of forces, the intensity of a force is represented by a straight line, while the direction of the same line

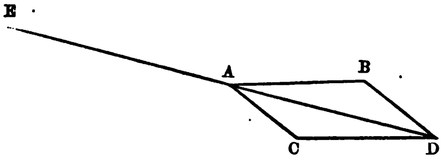
indicates the direction in which the force acts. Thus if AB represent a force acting on B, the length of the line AB will measure the intensity of the force; if it be

a pulling force acting from B towards A, it is called the force BA; if a pushing force acting from A towards B, it is called the force AB; the order of the letters serving to show the direction of the force. A pulling force takes the name of a tension; a pushing force that of a thrust.

3. It is necessary to enunciate at once what is called the principle of the transmission of force; which is, that a force may be conceived to act anywhere in its line of action without altering the effect. Whether the force AB acting upon the point B is placed at A or B, or anywhere between, will obviously make no difference, so long as its intensity and direction remain the same.

4. The simplest case of balanced forces is when two forces of equal magnitude act upon the same point in precisely opposite directions. This requires no explanation. But if three or more forces act upon a point, we must show what are the conditions of equilibrium. Two forces can only balance each other if they are equal and opposite. But there are various positions in which three or more forces may balance each other, according to their respective intensities. Equilibrium between three forces will clearly take place if the resultant, as it is called, or joint effect of two of the three forces is equal in intensity, and acts in the opposite direction, to the third force.

It is necessary therefore to know how to find the resultant of two forces acting upon a point, which is thus accomplished. Let AB, AC represent two forces acting upon the point A. Complete the parallelo-

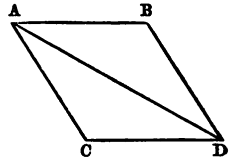


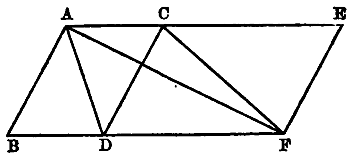
gram ABCD, and draw its diagonal AD. AD will represent both in magnitude and direction the resultant or joint effect of AB and AC, so that if DA be produced till AE=AD, the forces represented by the three straight lines AB, AC, AE, acting upon the point A, will keep each other in a state of equilibrium. This proposition is known as the Parallelogram of Forces. Various proofs of it have been given; but one only, that of Duchayla, is adapted to an elementary treatise like the present. It is as follows.

5. The two parts of the proposition, that which concerns the *direction* and that which concerns the *magnitude* of the resultant, are in this proof treated separately. First then we will demonstrate that if AB, AC represent in direction and in magnitude two forces acting upon the point A, their resultant or joint effect will act in the direction AD, which is the diagonal of the parallelogram ABCD described upon AB and AC.

6. If the two forces are equal to each other, there

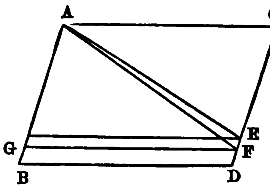
will be no reason why their resultant should incline more towards the direction of one than that of the other; its direction will therefore bisect the angle made by the directions of the two components; and as the diagonal of a parallelogram whose sides are equal bisects the angle made by the sides, that direction evidently coincides with that of the diagonal of the parallelogram whose sides represent the forces. The proposition is therefore true when the two forces are equal. To extend it to the case of unequal forces, we begin by showing that if it be assumed to be true for two unequal forces, such as P and Q, and also for two others, P and R, it will also hold true for P and a force Q+R equal to the sum of the two others. Let AB, AC represent the two forces P and Q acting upon A; complete the parallelogram ABCD, and by the assumption (made provisionally), AD, its diagonal, will represent the direction of the resultant of P and Q. By the principle of the transmission of force, a force may be supposed to act anywhere in its line of direction; we may therefore suppose the resultant of P and Q, which acts in the direction AD, to act at the point D; and since the resultant of two forces is in all respects equivalent to its components, we may suppose P and





Q themselves to act at D, P parallel to AB its own direction, and Q parallel to AC; or, using once more the principle of the transmission of force, P may be supposed to act at C in the direction CD. Now let AB and CE represent in direction and magnitude two other forces P and R acting upon A; R may be supposed to act at C, and it has just been shown that P may be also supposed to act there in the direction CD; by the assumption, therefore, the resultant of P and R will be represented in direction by CF, the diagonal of the parallelogram CEDF; hence, reasoning as before, the resultant of P and R may be supposed to act at F, or P and R themselves may be supposed to act at that point parallel to their original directions. The force Q also, which we have supposed to act at D in the direction DF, parallel to AC, may be supposed to act at F in the same direction. Forces therefore represented by P and Q+R acting at A, may be supposed to act at F, or F is a point in the line of action of the resultant of P and Q+R; AF is therefore the direction of that resultant. If, then, the proposition is true for P and Q, and also for P and R, it is true for P and Q+R. But it is true for P and P (two equal forces), and also for P and P; therefore it is true for P and P+P or 2P; therefore for P and P+2P or 3P; therefore for P and *m*P, where *m* is any whole number. And in like manner it may be shown to be true for *m*P and *n*P, where *m* and *n* are any two whole numbers. Lest there should be any doubt whether it is true when the ratio of the two forces cannot be thus represented by two whole

numbers, we will show that the contrary supposition involves an absurdity. Let AB, AC represent two such forces (incommensurable as they are called); complete the parallelogram A B C D, and if AD, its diagonal, is not the direction of the resultant, let AE be that

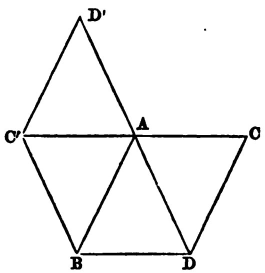


direction. Let AC be divided into a number of equal parts, each part less than ED, and set off distances of the same length along CD, beginning at C; one of the divisions must fall between E and D; let it fall at F, and complete the parallelogram AGFC. Then AF, the diagonal of that parallelogram, will be the direction of the resultant of the forces AG, AC, which are commensurable. But AF makes a larger angle with AC than AE does; that is, the resultant of AG and AC lies further from AC than the resultant of AB and AC, although AG is less than AB; which is absurd. AE therefore is not the direction of the resultant of AB and AC; and it would be equally easy to show that no line except AD is that direction.

7. We will now come to the other part of the proposition, and proceed to show that if AB, AC represent in direction and in magnitude two forces acting upon the point A, their resultant or joint effect will be represented in magnitude, as well as in direction, by AD, the diagonal of the parallelogram ABCD described upon AB and AC.

8. Produce DA to D′, making AD′ equal to the re-

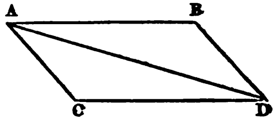
sultant of AB and AC in magnitude; complete the parallelogram ABC′D′ and join AC′. Then, since AD′ is equal to the resultant of AB, AC, and drawn in the direction opposite to that of their resultant, the three forces AB, AC, AD′ will balance one another, and consequently any one of them is in the direction of the resultant of the other two; therefore AC is in



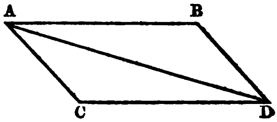
the direction of the resultant of AB and AD′, and AC′ being also in that direction (being the diagonal of the parallelogram formed upon AB, AD′), AC, AC′ are in the same straight line. Hence ADBC is a parallelogram; therefore AD = BC′; but BC′ = AD′; therefore AD = AD′. Since, then, AD′ was made by the construction equal in magnitude to the resultant of AB and AC, it follows that AD is also equal to that resultant in magnitude; as was to be proved.

9. We have thought it necessary to demonstrate this important proposition, because it is the foundation of the whole theory of statics. We will repeat, for the sake of clearness, the enunciation given in § 4. If two straight lines meeting in a point represent in direction and magnitude two forces acting upon that point, and a third straight line equal to the diagonal of the parallelogram formed upon the preceding two and drawn by producing that diagonal backwards, represent a third force acting upon the same point,

the three forces will keep each other in equilibrium. This may be easily placed in another form of great convenience, in which it is known as the Triangle of Forces. Since AB, AC, and DA represent in direction and magnitude three forces which keep each other at rest, and AC= BD, the three forces are represented by the three sides of a triangle ABD, taken in the order of the figure AB, BD, DA. And since two triangles, all whose sides are respectively parallel each to each, are equiangular, and therefore (Euc. vi. 4) have their sides about the equal angles proportionals, it follows that if three forces, acting in the same plane, be in equilibrium upon a particle, and if in that plane we draw any three straight lines parallel to the directions of the forces, then the three sides of the triangle so formed, taken in order, will be in the same proportion as the forces.

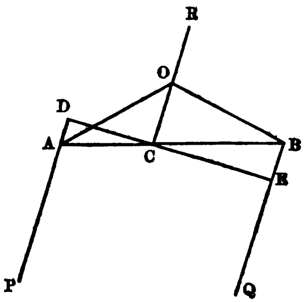


10. As we may substitute for two forces their resultant, so may we substitute for any force considered as a resultant two other forces which may be considered as its components. And it results from the foregoing that for any force there may be substituted as equivalent any two others, lines representing which others can be formed into a triangle with a line representing it; the two acting in the direction of the other sides



of that triangle, but taken in the reverse order to that of the original force. For AD in the figure may be substituted AB and AC, which produce precisely the same effect; but AC is equal and parallel to BD; therefore for AD may be substituted as equivalent AB and BD, which form a triangle with it. This is called resolving the force AD into the directions AB, BD.

11. The plan of this short treatise only admits of our treating of the simplest cases of balanced forces; and we will now show what are the conditions of equilibrium when three forces do not act immediately upon a point, but two of them acting at the ends of a rigid rod, it is desired to know where the third must be placed in order that no motion may ensue. To simplify the problem, we shall suppose all three to act in parallel directions, two in the opposite direction to the third, which must, of course, in respect to magnitude, be equal to the sum of the other two. AP, BQ, CR then will represent the forces, AB the rigid rod, and C the point in it at which the force CR, equal to the sum of AP and BQ, must act in order to keep them in equilibrium. At A and B suppose two



equal forces to act in the contrary directions CA, CB, which of course will not affect the equilibrium. The

resultant of BQ and that force will act in a direction such as that represented by OB; and the resultant of AP with the equal force in such a direction as OA; both therefore may be supposed to act at the point O, where those two directions intersect. The two equal forces neutralizing each other, a resultant remains equal to the sum of AP and BQ, or to CR, acting in the direction OC parallel to the three original forces. The point O therefore lies in the line CR. Now let P, Q, and S denote the forces AP, BQ, and the two equal forces which were applied in opposite directions at A and B. Then the three sides of the triangle AOC, taken in order OC, CA, and AO, are respectively in the directions of the forces P, S, and a direction opposite to their resultant, or the same as that of a force that would balance them; it follows from the triangle of forces that P : S : : OC : CA. In like manner, by considering the triangle BOC, we obtain Q : S : : OC : CB. From these two proportions it is easy to deduce P : Q : : CB : CA, or $P \times CA = Q \times CB$.

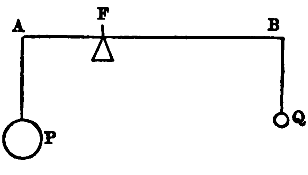
12. If DE be drawn perpendicular to AP or BQ, the triangles ADC, BEC will be similar, and consequently AC : CD : : BC : CE; so that $P \times CD = Q \times CE$. We conclude therefore (and in this form the proposition may be generalized, being applicable whatever be the directions of the forces) if a force acting upon a rigid rod keep two other forces so acting in equilibrium, its point of application must be such that the product of either force, multiplied by the perpendicular drawn from that point upon its direction, will be equal to each other.

13. The point of application of the third force is called the fulcrum, and the above proposition is usually expressed by saying that the moments of the other two forces about the fulcrum are equal to each other; the moment of a force with respect to a point being defined to be the product of that force into the perpendicular distance from the fulcrum to the line of direction of that force.

14. Here we have the principle of the Lever, which is of so great practical importance as a mechanical power. The weight of a body is the force by which it tends (*i. e.* the sum of the tendencies of all the particles of which it is composed) *towards* the centre of the earth; or in the direction which we call downwards, while the opposite direction *from* the earth's centre takes the name of upwards. If, then, two weights be suspended at the ends of a rod, one point of which is kept at rest as a fulcrum by a support capable of exerting a force sufficient to sustain the sum of the two weights, the principle just enunciated is plainly applicable, and the fulcrum must be so situated that the products of its distance from the point of suspension of either weight, multiplied by the numerical measure of that weight, shall be equal to each other; these products being the moments of the weights about the fulcrum. This may be further extended; and if several weights be suspended at various distances on each side of the fulcrum, the sum of the moments of the weights on one side will be, if equilibrium is maintained, equal to the sum of the moments of those on the other side.

15. Before proceeding further, we shall notice one or two practical applications of the lever in the form we have described it, which is commonly called that of a lever of the first kind.

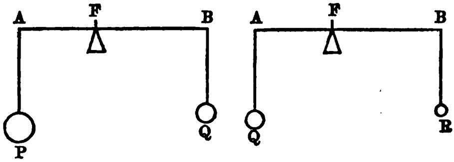
16. If the two arms of the lever are equal in length, it is manifest that to produce equilibrium the two weights must also be equal to each other. And this is the principle of the balance or pair of scales, a weight being measured by placing it on one side, so as just to balance some other known weight on the other side. It is therefore essential to the accuracy of the instrument that the fulcrum be equidistant between the points of suspension of the two weights, *i. e.* that the two arms of the balance be equal in length. Nevertheless, if the distances of the fulcrum from those points be unequal, provided that both are known, there will be no difficulty, though more trouble, in ascertaining the value of one weight by balancing it with another. For let AF, BF be the two arms of the balance, F being the fulcrum, and P, Q the two weights, of which the value of P is known, and that of Q is sought by making P balance it in the scales. We



have $P \times AF = Q \times BF$; therefore $Q = \frac{P \times AF}{BF}$, which equation determines Q if AF and BF are known.

17. Even if they are not known, it will be possible to determine Q by the following process. After find-

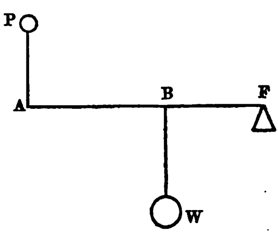
ing a weight P which will balance it as before, transfer Q to the other side of the scales, and find a weight R



which will balance it in this new position. Then we have the two equations, $P \times AF = Q \times BF$ and $Q \times AF = R \times BF$. From this it is easy to deduce $Q^2 = P \times R$, or $Q = \sqrt{P \times R}$.

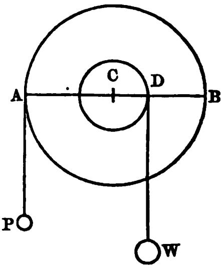
18. The steelyard differs in its mode of application from the ordinary balance; in it the substance to be weighed is placed in the short arm, and a known weight is moved along the other arm until equilibrium is effected; the distance of the known weight from the fulcrum, compared with that of the substance to be weighed, gives the required ratio of the two weights.

19. When the fulcrum is not between the two weights, the lever is said to be of a different kind. Thus, let the fulcrum be at one end, and a power P be exerted vertically upwards to raise a weight W; we shall have, as before, if the power just sustain the weight, $P \times AF = W \times BF$. In this arrangement the lever is of the second kind. The third

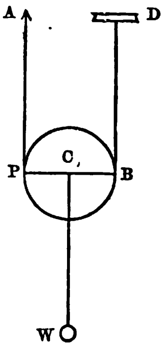


kind is when the power is between the weight and the fulcrum.

20. We shall now say a few words about the mechanical power called the Wheel and axle, which is merely a modification of the lever. In this instrument the weight is fixed to a rope, which is wound round a small cylinder called the axle, and the power is suspended in the same way on a larger cylinder turning on the same centre, which is called the wheel. The power and the weight then act as in a lever, of which C is the fulcrum, and CA, CD are the arms. To produce equilibrium we must have $P \times AC = W \times CD$. It is evident that no difference will ensue if the power is applied at the end of a spoke, and in any part of the circle, provided it be exerted in the direction of the tangent to the circle at the point of application.

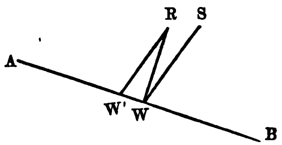


21. The Pulley, in its simplest form, consists of a wheel moveable round its axis, which is itself moveable. A cord is passed round the wheel, one end of which is fixed as at D, while the power P by its tension at the other end keeps the cord stretched; the weight W is suspended from the centre of the pulley.



Now, here the weight is sustained by the tension of the string acting at B, and of the power at A. The string being kept stretched, these two tensions are equal; either therefore is equal to the power P, and the sum of the two to 2P. If they are kept in equilibrium by the weight W acting in the opposite direction, we must then have $W = 2P$, or $P = \frac{1}{2}W$.

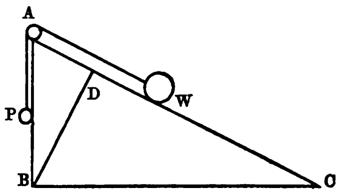
22. The only other mechanical power we can treat of with advantage here is the Inclined Plane, the conditions of equilibrium on which, as on the lever, admit of an easy investigation by the Triangle of Forces. It is necessary, in the first place, to show in what direction the resistance of an inclined plane to a body placed upon it acts, and it is not difficult to prove that this must be in a direction perpendicular to the surface of the plane. For let AB be the inclined plane, and W a weight placed



upon it, which is thus restrained from moving in any other direction to a lower place, but (the plane being supposed perfectly smooth) is free to run down the length of the plane. Now we say that the resistance offered by the plane is in the direction WR, perpendicular to AB. If not, let it be in some other direction, as WS or W'R parallel to it, and let its value be represented by the line W'R. Then, by the principle of the resolution of forces (§ 10), W'R may be resolved into two, W'W and WR, at right angles to each other. But W'W would be a force urging the body along the plane, which the resistance of the

plane cannot exert; hence W′W is equal to nothing, and the only force left is in the direction WR.

23. We are now in a position to discuss the conditions of equilibrium when a body rests upon an inclined plane. Let, then, a weight W be so sustained upon the inclined plane ABC, by means of a power P, the pressure of which acts vertically downwards, but is changed into a force exerted in the direction CA by means of a cord passed over a fixed pulley* at A. The weight W then is kept at rest by three forces; its own weight acting vertically downwards, the tension of the power P acting in the direction CA, and the resistance of the plane, acting perpendicularly to its surface or in the direction BD, which is at right angles to AC. Since then the three sides of the triangle BAD, taken in order, AB, DA, BD, are parallel to the directions of these forces which keep each other in equilibrium, they are also, by the Triangle of Forces, proportional in magnitude to those forces respectively. Hence P : W : : DA : AB. Now we have by Euclid vi. 8, DA : AB : : AB : AC. Consequently P : W : : AB : AC, or the power is to the weight which it thus sustains on an inclined plane as the height to the length of the plane.



24. Having thus treated with great brevity of the most important mechanical powers, and the condi-

* The only effect of a *fixed* pulley is to change the direction of a force.

tions of equilibrium on each, we shall conclude what we have to say on statics by a few words concerning what is called the centre of gravity of different bodies.

25. In treating of the principle of the lever, it was shown that if a rigid rod, loaded with a weight at each end, was supported at a point whose distances from the two ends were inversely as the weights suspended from them, the moments of the two weights about that point would be equal, and the rod would rest, being in a position of equilibrium. In like manner, there is a point in every body or system of bodies rigidly connected together, which, if it be supported, the body or system will balance about it in any position. That point, which is called the centre of gravity of the body or system of bodies, requires for its support a force equal to the weight of the whole body or system, which may, so far as effects are concerned, be considered to be all collected together at its centre of gravity.

26. From this it results that the centre of gravity of every body always takes up the lowest position it can reach; if it is sustained, the body will be supported; if not, the body will twist about it and take up a new position, where its weight acting at its centre of gravity is counteracted by an equal opposing pressure below. When, therefore, a body is placed on a horizontal plane, with which it is in contact by its lower part or base, if the centre of gravity of the body is vertically over a point within the base, the body will be in a position of equilibrium, and will rest undisturbed; but if a vertical line through the

centre of gravity fall without the base, the body will fall. (It is to be understood that the effects of friction are wholly neglected here, and all bodies supposed perfectly smooth.)

27. When a body is suspended so that its centre of gravity is below the point of suspension, and is in a state of equilibrium with the centre of gravity at the lowest point, or in the same vertical line with the point of suspension, if it be disturbed from that position, it will return to it by the force of gravity. Such a position, therefore, is called one of *stable* equilibrium.

28. But when a body is supported on a small base, so that its centre of gravity is above the point by which it is supported, and is in a state of equilibrium with the vertical line through the centre of gravity passing within the base, if it be more than very slightly disturbed from that position it will not return to it, but will move until the centre of gravity has taken up a new position as low as it can reach. Such a position is therefore called one of *unstable* equilibrium.

Section II.

MECHANICS.—DYNAMICS.

1. Dynamics, in its full extent, is a branch of mathematics involving analysis of the highest order, and discusses the general problem of the effects of

several forces acting at once upon bodies either at rest or in motion, in producing motion in them, or in changing the motion they already have. All we propose to do in this short sketch of its first principles, is to explain as simply as possible the fundamental laws by which all motions in bodies are regulated, and to give some simple illustrations of their effects.

2. Matter is of itself indifferent to rest or motion. If a body is at rest, and no external force is brought to bear upon it, it must for ever remain at rest; if by any force it is set in motion in any direction, and neither that nor any other force afterwards acts upon it, it must for ever continue to move in that direction with the same velocity which had been impressed upon it when it was last left to itself. This property of bodies is known by the name of their *vis inertiæ*—a term implying that they have no power either to set themselves in motion, or to avoid moving if any force, however small, is brought to bear upon them, or to cease or change their motion if once so put in motion by a force.

3. The first law of motion then is this: that if a body is acted on by no external force, it will be either at rest, or in uniform motion in a straight line. So that if a body is either at rest or in uniform motion in a straight line, it may be concluded that it is acted upon by no external force, and that it has been in identically the same state since the time when an external force last acted upon it.

4. Though this law cannot, by the nature of the

case, be directly proved by experiment, yet it may be inferred with a probability so strong as almost to amount to demonstration from experiments which we daily witness. That bodies are unable to put themselves in motion needs indeed no proof; and when they are put in motion by an external force, we see that that motion is continued to a greater distance, and ceases more gradually the fewer and slighter are the causes (such as the resistance of the air or other medium through which the motion is performed, or the friction of the body against the ground, or any surface with which it is in contact during its motion) which tend to check, retard, and gradually stop its motion.

5. From what we have said, it is at once apparent that if a force acts continuously upon a body, it will generate in that body a constantly increasing motion. For if the force ceased to act, the body would continue to move in the same direction and with the same velocity as it was moving at the moment of that cessation; but the force continuing to act, the body possesses after any given moment the uniform motion produced by the force having acted upon it *up to* that moment, and also the motion produced by the continued action of the force *after* that moment.

6. Forces, then, from this point of view are divided into impulsive, and constant or accelerating forces; and it is necessary clearly to understand what is meant by each of these.

7. No effect in nature is absolutely instantaneous;

and a force, in order to produce any effect upon a body's motion, must act upon it during *some* duration of time. But that duration may be too short to be cognizable by our senses, and then we can only estimate the force by the absolute effect produced when it has ceased to act. Such a force is called an impulsive force, and the only motion it can ever be seen to produce is a uniform motion, *i. e.* a constant velocity, because we are not able to consider it in the act of producing motion, but only after it has produced it, which motion must therefore be uniform by the first law of motion, derivable directly from the vis inertiæ, which is the universal property of bodies. This then may be expressed by the equation $s=vt$, where s represents the space described in the time t with the velocity v.

8. When, on the contrary, the same force acts upon a body during an appreciable interval of time, it will generate in that body a certain velocity in a certain interval of time (according to its intensity), and in the next equal interval of time the same velocity, so that at the end of the two intervals of time, double the velocity will have been generated of that which was generated at the end of the first interval, and so on; the force, being constant, generating in each successive interval of time an equal velocity, and therefore producing a constantly increasing velocity by equal increments; and the velocity produced at any moment being proportional to the time which has elapsed up to that moment since the force began to act, and the force being

measured by the velocity it has generated, not absolutely, but at the end of a given unit of time. Such a force is called a constant, or an accelerating force.

9. The most familiar instance of such a force is the force of gravity, as it is called. We shall say something about this force generally hereafter, but shall here define it by its manifest effect on the earth's surface, as a constant force by which all bodies tend to move, and if they are not supported, *i.e.*, if that tendency is not counteracted by some other force (as referred to in Statics), actually *do* move downwards or towards the centre of the earth. And we shall illustrate the above principle of the action of a constant force by the effects of gravity, which may also be of practical application.

10. We must first give the units usually adopted of time and space. These are one second and one foot. The velocity expresses the number of units of space described in one unit of time; so that a velocity of 10 implies that a space of 10 feet is described in one second of time, or at least that that space would be described in one second of time if the velocity continued uniform for that second.

11. By the definition, then, of constant force we have the equation $v=ft$, where f represents the force under consideration, measured by the velocity it produces in the unit of time. And we must show how in this case to determine s, the space described in the time t, with this constantly increasing velocity.

12. When the force in question is the force of gravity, the letter g is used to denote it, so that we

have $v=gt$. Now it has been ascertained by numerous experiments that the force of gravity on the earth's surface is of such an intensity as to generate in a body subjected to it a velocity downwards or towards the earth's centre of about 32·2 feet in one second of time. This then is expressed by saying that $g=32{\cdot}2$, or $v=32{\cdot}2\times t$ is an equation applicable to all cases of bodies surrendered freely to the action of gravity.

13. The equation $s=vt$ is of course only applicable when the velocity is constant, but when the velocity is continually increasing under the action of a constant force, the space described in a given time will be the mean of that which would have been described if the velocity had been during the whole interval what it was at the *beginning* of it, and that which would have been described if it had been during the whole interval what it was at the *end* of it. Thus if the time be reckoned from the body's being in a state of rest, at the beginning of that time the velocity will be nothing; let us call the velocity at the end of it V, and the mean velocity will be $\frac{1}{2}(0+\text{V})=\frac{1}{2}\text{V}$, so that $s=\frac{1}{2}\text{V}t$. And since the velocity is always equal to the product of the force into the time, we have in the case of a constant or accelerating force, $s=\frac{1}{2}ft\,.\,t=\frac{1}{2}ft^2$.

14. This proposition is so important and fundamental that it will be proper to give a strict demonstration of it. Let then the time t be supposed to be divided into n intervals, each equal to τ, so that $n\tau=t$, then the velocities at the *beginnings* of the

1st, 2nd, 3rd, 4th ... *nth* of these intervals will be

$$0,\ f\tau,\ 2f\tau,\ 3f\tau\ \ldots\ (n-1)f\tau,$$

and at the *ends* of the same intervals

$$f\tau,\ 2f\tau,\ 3f\tau,\ 4f\tau\ \ldots\ nf\tau.$$

Then, if the body were to move during each successive interval of τ with the velocity which it had at the *beginning* of that interval, the space described would be (adding the successive spaces, each obtained by the principle $s=vt$)

$$0\,.\,\tau+f\tau\,.\,\tau+2f\tau\,.\,\tau+\ldots(n-1)f\tau\,.\,\tau,$$

which is $=f\tau^2\{1+2+3+\ldots+(n-1)\}$

$$=\frac{n-1}{2}\,.\,n\,.\,f\tau^2=\tfrac{1}{2}n(n-1)f\tau^2.$$

Now $n\tau=t$, consequently $\tau^2=\dfrac{t^2}{n^2}$; and substituting this value for τ^2, we reduce the above sum to

$$\tfrac{1}{2}ft^2\,.\,\frac{n(n-1)}{n^2}=\tfrac{1}{2}ft^2\left(1-\frac{1}{n}\right).$$

But if the body were to move during each successive interval of τ with the velocity which it has acquired at the *end* of that interval, the space described would, similarly determined, be

$$f\tau.\tau+2f\tau.\tau\ \ldots\ldots+nf\tau.\tau$$

$$=f\tau^2(1+2+3\ \ldots\ldots+n)$$

$$=\frac{n}{2}(n+1)f\tau^2$$

$$=\tfrac{1}{2}f\tau^2.\,n(n+1)$$

$$=\tfrac{1}{2}ft^2.\,\frac{n(n+1)}{n^2}$$

$$=\tfrac{1}{2}ft^2\left(1+\frac{1}{n}\right).$$

Now, since the actual velocity is never constant, but is continually increasing during the whole duration of t, the space *actually* described by the body will be intermediate between those that would be described under each of the above two hypotheses; that is, s lies between

$$\tfrac{1}{2}ft^2\left(1-\frac{1}{n}\right)$$

and $$\tfrac{1}{2}ft^2\left(1+\frac{1}{n}\right),$$

however large n be taken, or into however many intervals the time be supposed to be divided. But when n becomes indefinitely large, each of these two quantities becomes $\frac{1}{2}ft^2$; and therefore s, which always lies between them, must coincide with them in the limit, that is,

$$s=\tfrac{1}{2}ft^2.$$

15. The spaces then described in one, two, three, four, &c. units of t being

$$\tfrac{1}{2}f\times1,\ \tfrac{1}{2}f\times4,\ \tfrac{1}{2}f\times9,\ \tfrac{1}{2}f\times16,\ \&c.,$$

those described in the successive units will be

$$\tfrac{1}{2}f\times1,\ \tfrac{1}{2}f\times(4-1),\ \tfrac{1}{2}f\times(9-4),\ \tfrac{1}{2}f\times(16-9),\ \&c.,$$

or $$\tfrac{1}{2}f\times1,\ \tfrac{1}{2}f\times3,\ \tfrac{1}{2}f\times5,\ \tfrac{1}{2}f\times7,\ \&c.,$$

that is, in the proportion of the odd numbers

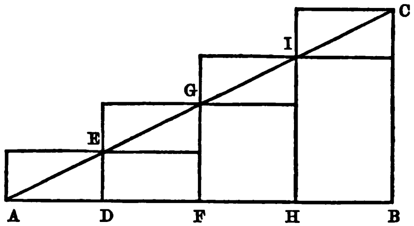
1, 3, 5, 7, &c.,

forming an arithmetical progression whose common difference is $\frac{1}{2}f\times2$, or f.

16. Since $v=ft$, $v^2=f^2t^2=2fs$; therefore $v=\sqrt{2fs}$.

17. The proposition proved in § 14 is so important that we will endeavour to make it clear by a geometrical illustration also. Since equal velocities are

generated by a constant force in equal times, or, which is the same thing, the velocities generated are as the times in which they are generated, any number of velocities and their corresponding times of production may be represented by the homologous sides of a series of similar triangles. Thus, if DE, FG, HI be parallel to BC, and AD, DF, FH, HB



represent successive intervals (such as seconds) of time, and DE be in such proportion to AD that it will numerically measure the velocity generated by the force under consideration in one second, then will FG, HI, BC, &c. represent in the same way the measure of the velocity generated in two, three, four, &c. seconds. Now form about AE, EG, GI, IC as diameters, parallelograms as shown in the figure. Then if the body be supposed to move during the whole of each successive second with the velocity which it has at the beginning of that second, the whole space passed through in four seconds will be represented by the triangle ABC *minus* the halves of those parallelograms; since in each second it will be (as with a uniform velocity we must have $s=vt$, so that the space may be represented by the rectangle formed upon the velocity

and time) measured by a point or nothing, parallelogram EF, parallelogram GH, parallelogram IB, &c. But if the body be supposed to move during the whole of each successive second with the velocity which it has at the end of that second, the whole space passed through in four seconds will be represented by the triangle ABC *plus* the halves of the same parallelograms; for in each second it will be measured by the parallelograms AE, DG, FI, HC. Now, as in the actual fact, when the force is constant, the velocity is continually increasing, not only from second to second, but from one infinitesimally small fraction of a second to the next, we shall more nearly represent it the more we divide AB into spaces representing smaller intervals of time; the consequence of which will be that the parallelograms about AC will become smaller till they become insignificant and finally vanish altogether; so that the actual space passed through in a duration of time represented by AB with a force capable of generating in one second a velocity represented by DE, will be measured by the triangle ABC itself. But as the area of a triangle is equal to half the product of its base and altitude, this will be $\frac{1}{2}$ AB$\times$BC; or s, the space, will be $=\frac{1}{2}vt$; or, since $v=ft$, we have as before $s=\frac{1}{2}ft^2$.

18. Since v represents the velocity generated at the end of the time t, we have this consequence deducible from our formula: the space described by a body falling from rest in any given time is equal to one-half of that which would have been described

had the body fallen during the whole time with the velocity which it had at the end of it. For the space described with a constant velocity v is $=vt$, and we have just shown that when a body falls from rest $s=\frac{1}{2}vt$, where v denotes the velocity acquired at the end of the time represented by t.

19. It will be well here to give an example or two of the motion of falling bodies, by means of the formulæ we have established, which are in this case $v=gt$ and $s=\frac{1}{2}gt^2$, where $g=32{\cdot}2$ feet, and t is expressed in seconds of time.

20. *Ex.* 1.—Find the space descended in seven seconds by a body falling freely, and the velocity acquired at the end of that time.

Here $s=\frac{1}{2}gt^2=16{\cdot}1\times49=788{\cdot}9$ feet, the space descended.

And $v=32{\cdot}2\times7=225{\cdot}4$, the velocity acquired at the end of the seven seconds.

21. *Ex.* 2.—Find the time occupied in descending 400 feet.

$$\text{Here } t=\sqrt{\frac{2s}{g}}=\sqrt{\frac{2s}{32{\cdot}2}}=\sqrt{\frac{s}{16{\cdot}1}}=\sqrt{\frac{400}{16{\cdot}1}}$$
$$=\sqrt{24{\cdot}84}=5{\cdot}0 \text{ seconds.}$$

22. *Ex.* 3.—If a body fall freely for 5 seconds, how far will it fall during the last, or fifth second?

If in the formula $s=\frac{1}{2}gt^2$ we put successively 5 and 4 for t, we shall obtain the difference between the spaces fallen in five and four seconds, which will be the space fallen in the fifth second. It will be, therefore, $\frac{1}{2}g\times25-\frac{1}{2}g\times16=16{\cdot}1\times9=144{\cdot}9$ feet.

23. *Ex.* 4.—If a body falls 176 feet during the

last second of its fall, what was the height from which it fell?

Here it will be best first to find the whole time of descent; and since the space fallen in any second is $=\frac{1}{2}gt^2-\frac{1}{2}g(t-1)^2$, where t is the number of seconds which at the expiration of that second has elapsed since the commencement of the fall,

$$=\tfrac{1}{2}g\,(2t-1)=32{\cdot}2\times t-16{\cdot}1\,;$$

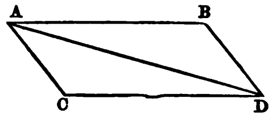
if we add 16·1 to the space given and divide the sum by 32·2, we shall have the time of the whole descent, which is therefore $\frac{176+16{\cdot}1}{32{\cdot}2}=6{\cdot}0$. So that the body has fallen for six seconds, in which time it must have descended $16{\cdot}1\times 36=579{\cdot}6$ feet.

24. We will now proceed to enunciate the second law of motion, which is concerned with two or more forces acting simultaneously upon a body, and asserts that, when this is the case, each force produces its whole effect in generating a velocity in its own direction as if the others had no existence. The following are simple experiments illustrating this law:— If a ship be sailing with a uniform motion, it will require the same force to throw a ball from the bow to the stern as from the stern to the bow, or the same distance in any other direction; and if it be dropped from the top of a mast, it will fall at the foot of the mast. So also the motions of a person on the deck of a vessel moving uniformly in a straight line, or on the floor of a railroad-carriage in equable motion, are quite independent of the motion of the ship or the carriage; if he jumps upwards, he will

descend upon the very place from which he rose, and he can walk in any direction, precisely as if he were standing upon fixed ground.

25. We will particularize the three cases of two forces acting upon a body, (1) when both of them are impulsive forces; (2) when one is impulsive and the other constant; and (3) when both are constant.

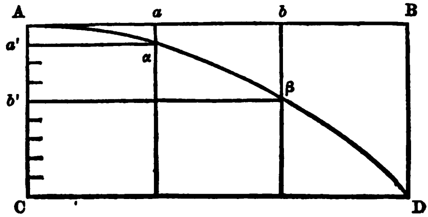
26. If an impulsive force acts upon a body at A, such as, if acting alone, would produce in it a uniform motion from A to B in a certain time, and at the same moment another



impulse acts upon it, such as, if acting alone, would produce a uniform motion from A to C in the same time, the body will, under the joint influence of the two impulses, describe in that time the straight line AD, which is the diagonal of the parallelogram whose sides represent the spaces that would be described by the force of each impulse separately. This is called the parallelogram of velocities; and sometimes the dynamical parallelogram of velocities, to distinguish it from the geometrical, which is merely a geometrical convention for resolving the motion of a body in one direction into two directions, making any angle with each other, similar to what we showed in treating of balanced forces (Statics, § 10); thus a velocity represented by AD may be resolved into velocities in the directions AC and CD, represented by the lines AC, CD respectively.

27. But if an impulsive force has acted upon a

body so as to cause it to move uniformly in the direction AB, while at A a constant or accelerating force begins to act upon it in the direction AC, the resulting motion will evidently not be in a straight line. We will suppose, to simplify the case, that the constant force acts always in the direction AC, or in a



direction parallel to it, or that it acts, in so far as direction is concerned, in the same manner as an impulsive force would have acted. Now if AB be divided into any number of (say three) equal parts, A*a*, *ab*, *b*B, the body would describe, under the action of the impulsive force alone, these three spaces in three equal spaces of time, as seconds for instance. But we have shown (§ 14) that the spaces described in successive seconds under the influence of an accelerating force, are as the odd numbers 1, 3, 5, 7, 9, &c.; in three seconds, therefore, they will be in the proportion of the three numbers 1, 3, and 5. Divide AC into nine equal spaces; let A*a'* be equal to one of them, *a'b'* to three, and *b'*C to five; then, the constant force being supposed to be of such an intensity as to generate in one second a velocity sufficient to carry the body through two of the spaces in a second,

it would, under the action of the constant force alone, describe these three spaces, Aa', $a'b'$, and b'C in three successive seconds. Under the influence, therefore, of the two forces jointly, the body will, at the end of each second, be at the points α, β, D; Aα being the parallelogram formed upon Aa, Aa'; Aβ that upon Ab, Ab', and AD that upon AB, AC. It is evident, therefore, that the body's motion will be in a curved line A$\alpha\beta$D. It will be, in fact, the curve known to mathematicians under the name of the parabola, one of the conic sections; but we are not at liberty to enter further into this, since in this little treatise we do not suppose the reader to be acquainted with the doctrine of the conic sections. But we may mention, that as the earth's gravity, for the small parts of the earth's surface with which our experiments are concerned, may, although really tending towards the earth's centre, be considered as acting in parallel lines, this is the way in which a body, projected in any direction by any force on the earth's surface, actually *does* move,—that is, if we neglect the effect produced by the resistance of the air, and all other causes of disturbance.

28. In a similar way it would be easy to illustrate the motion of a body moving under the joint action of two accelerating forces. If these begin to act upon it at the same instant, its path will be a straight line, since both will tend to accelerate it proportionally in equal times; but if they begin to act upon it at different moments, the direction of the path will be more or less changed from that of a straight line.

29. The simplest case of two forces acting simultaneously upon a body is when they both act in the same straight line; *i. e.*, either in precisely the same direction or in precisely opposite directions. The whole effect of both forces upon the body will, in the first case, be to produce the sum of the motions which either alone would have produced in the same direction; and, in the second case, to produce the difference of the motions which either alone would have produced. Although this is strictly on the principle of the second law of motion, yet in this instance it is so self-evident, that we have in effect made use of it in treating of the motions produced by a constant force acting alone; since the motion previously produced was dealt with as if it had been a uniform motion produced by an impulsive force, and added to the motion afterwards produced by the continuous action of the constant force.

30. The easiest illustration of this is when a body is projected by some impulsive force generating a constant velocity in the same direction as that in which the body is falling under the influence of gravity, or in the direction precisely opposite to it; *i. e.*, either directly downwards or directly upwards. Thus if a body is projected downwards with a velocity V, it will move in the time t by that force through a space $s = Vt$, and will fall in the same time by the action of gravity $s = \frac{1}{2}gt^2$. Under the joint influence therefore of gravity and the force of projection, it will fall in the time t a space measured by $Vt + \frac{1}{2}gt^2$. But if the body be projected upwards, or in the con-

trary direction to that of gravity, with the velocity V, it will move upwards in the time t through the space represented by the formula, $s=Vt-\frac{1}{2}gt^2$. Of this it will be advantageous to furnish an example or two.

31. *Ex.* 1.—If a body be projected vertically downwards with a velocity of 100 feet per second, how far will it fall in three seconds?

In the formula $s=Vt+\frac{1}{2}gt^2$, we have here $V=100$, $t=3$, $\frac{1}{2}g=16{\cdot}1$, so that s, the space fallen,

$$=100\times3+16{\cdot}1\times9=444{\cdot}9 \text{ feet.}$$

32. *Ex.* 2.—If a body be projected vertically upwards with a velocity of 100 feet per second, how high will it rise in three seconds?

Here we have to make the same substitutions as in the last example, but in the formula

$$s=Vt-\tfrac{1}{2}gt^2.$$

This gives us

$$s=100\times3-16{\cdot}1\times9=155{\cdot}1 \text{ feet.}$$

33. *Ex.* 3.—If an arrow be propelled vertically upwards from a bow with a velocity of 96·6 feet per second, how high will it rise, and how long will it be before it reaches the ground again?

In this example we have to consider when the velocities generated in two opposite directions become equal to each other and mutually destroy one another, thereby causing motion to cease. Now, the velocity generated in the arrow upwards by the force of the propulsion is constant, and is, by the supposition, 96·6; that generated by gravity downwards is constantly increasing by equal increments, and at any

time t is equal to gt, or $32{\cdot}2 \times t$. Motion upwards will therefore cease when $32{\cdot}2 \times t = 96{\cdot}6$, or when $t = \frac{96{\cdot}6}{32{\cdot}2} = 3$. Consequently the arrow will rise until the expiration of three seconds, and will then descend again, as if afterwards surrendered to the free action of gravity. Now, since it rises in any time t through a space $= Vt - \frac{1}{2}gt^2$, and when it rises until the opposing force of gravity will permit it to rise no longer, gt becomes $= V$, the space risen will be $gt^2 - \frac{1}{2}gt^2 = \frac{1}{2}gt^2$, or it will rise to the same height as it would have fallen in the same time under the free action of gravity; gravity retarding the velocity of projection of any body by the same stages as it would have produced the same velocity by its force of acceleration if the body had fallen from rest. The arrow will consequently rise to a height of $16{\cdot}1 \times 9 = 144{\cdot}9$ feet, and will then fall in the same space of time as that occupied in rising; the whole time of both ascent and descent taking up six seconds.

To make this destruction of motion *in one direction* more clear, we will exhibit the spaces passed through by the arrow in each of the successive six seconds.

In the first second it will rise

$$96{\cdot}6 - 16{\cdot}1 = 80{\cdot}5 \text{ feet};$$

in the second

$$96{\cdot}6 - 48{\cdot}3 = 48{\cdot}3 \text{ feet};$$

in the third

$$96{\cdot}6 - 80{\cdot}5 = 16{\cdot}1 \text{ feet};$$

in the fourth

$$96{\cdot}6 - 112{\cdot}7 = -16{\cdot}1 \text{ feet};$$

in the fifth

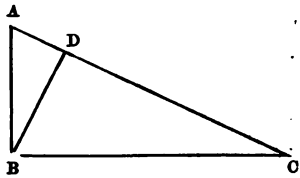
$$96{\cdot}6-144{\cdot}9=-48{\cdot}4 \text{ feet};$$

and in the sixth

$$96{\cdot}6-177{\cdot}1=-80{\cdot}5 \text{ feet.}$$

The — sign here implies that the arrow is falling instead of rising, and it is seen that the fall in the sixth, fifth, and fourth second is the same as the rise in the first, second, and third; the velocity upwards by the force of the propulsion continuing constant, but the accelerating force of gravity causing a larger and larger velocity downwards, so as first to balance and afterwards overcome the former.

34. We will now say something of the motion of a body on an inclined plane, when it falls by the action of gravity alone, but, instead of falling vertically, rolls down the plane. Let ABC be an inclined plane, AC being its length, AB its height, and BC its base. Now if AB be taken to represent the force of gravity, which is of course exerted vertically downwards, it is here, by the intervention of the plane not suffering it to produce its natural effect, resolved into the directions AD, DB. The part AB only produces a pressure on the plane itself; the part AD alone is effective in causing the body to roll down the plane. This part, then, is to the whole force of gravity as AD to AB, or (by similar



triangles) as AB to AC; that is, as the height to the length of the plane; so that the accelerating force of gravity, when acting upon a body on an inclined plane, is diminished in this proportion. If, then, we represent the height and length of an inclined plane by h and l, and write $g \cdot \frac{h}{l}$ for g in the formulæ previously demonstrated, we shall be able to solve all questions of motion on such a plane, of which we give the following example:—

35. *Ex.*—How far will a body descend from quiescence in four seconds on an inclined plane 400 feet long and 300 feet high?

Here

$$s = \tfrac{1}{2}g \cdot \frac{h}{l} \cdot t^2 = 16{\cdot}1 \times \frac{300}{400} \times 16 = 16{\cdot}1 \times 12 = 193{\cdot}2 \text{ ft.}$$

36. We have proved in § 15 that $v = \sqrt{2gs}$; so that on the inclined plane $v = \sqrt{\frac{2gsh}{l}}$. Now if the space descended be the whole length of the plane, this becomes $v = \sqrt{2gh}$. The velocity, therefore, acquired in falling down an inclined plane is dependent solely on the height of that plane, and is the same for all planes of equal height, however various may be their lengths; it is the same, in fact, as would be generated by falling freely through a distance equal to the height of the plane.

37. It also results from the formula $s = \frac{1}{2}g\frac{h}{l}t^2$ that when $s = l$, or the whole length of the plane is

descended, $t^2=\frac{2l^2}{gh}$; $\therefore t=l\sqrt{\frac{2}{gh}}$. If therefore h be constant, $\sqrt{\frac{2}{gh}}$ is a constant quantity, and t varies as l; that is, for inclined planes of the same height, the time of descent is exactly proportional to their length.

38. Hitherto we have spoken of the motion produced in bodies by a force or forces acting upon them, without taking account of the mass or quantity of matter contained in a body so moved. And it was unnecessary to do so in discussing the questions we have as yet considered. For the equation $v=ft$ (§ 11) resulted immediately from the first law of motion, and is applicable to all bodies, since (any quantity of matter being utterly indifferent of itself to rest or motion, and therefore compelled to yield to any force impressed upon it from without, and to move in the direction in which and with the velocity with which that force urges it) the continuation of the action of that force (supposed constant) continually adds to the motion previously produced, and thereby increases the velocity of the body. In speaking of falling bodies, we have stated the value which has been found by experiment of the force acting upon them by which they are caused to fall. Now this force resides in the earth, as is evident by the direction of the motion which is generated by it, and being exerted upon every particle of matter without exception, causes in each and every particle surrendered to its free influence the same amount of motion,

thereby producing the same velocity in every unsupported body, whether that body be large or small, heavy or light, *i. e.* contain a larger or smaller quantity of matter.

39. This truth, that the earth's attraction is the same upon every kind of matter, could not indeed have been known *à priori*; neither is it patent to the senses at first sight. Nay, we constantly see, in apparent contradiction to it, a heavy body fall to the ground more quickly than a light one, as if the earth exerted a greater attracting force upon the former than upon the latter. But more refined observations show that this is only because the heavier bodies, being denser, or containing matter more closely packed together, experience less resistance from the air, in which we ordinarily see them fall, than those which are lighter in proportion to their size, or contain the same quantity of matter spread through a larger space. For the more the space in which the fall takes place is void of air, the more nearly do various bodies fall through equal spaces in the same time, to however great a degree their weights may differ.

40. In treating, then, of the fall of bodies, we were able to neglect the question of what quantity of matter they might consist, the velocity of the fall being the same whether that is great or small—it being supposed either that the fall takes place in a space destitute of air, or that the body is sufficiently dense to fall without being sensibly retarded by the resistance of the air. But when by this, or by any other means, motion has been impressed upon a body,

that motion will be communicated to any other body with which it is directly or indirectly brought into contact, in an amount varying with the mass or quantity of matter of which the body so impelling another consists. For the *intensity* of a body's motion, or the velocity with which it moves, and the *quantity* of its motion are not the same, the latter being the sum of the intensity of the motion of all the particles of which it is composed, and therefore proportional in different bodies possessing the same velocity to the mass or quantity of matter which they contain. This quantity of motion of a body is called its momentum. So that the momentum is proportional to the product of the mass of a body and the velocity with which it is moving.

41. Now, if a body is brought either directly or indirectly (*i. e.* either with or without the intervention of other matter) into contact with another body, its moving force, or the quantity of motion which it is capable of communicating to that other body, is in the direct proportion of its own momentum or quantity of motion. This is the proposition which is now known under the name of the third Law of Motion. It is evident that the intensity of motion or velocity communicated will be inversely proportional to the mass or quantity of matter contained in the body so set in motion by another. For if M, m be the masses of the impelling and impelled bodies, and V, v their velocities, we must always have $M \times V = m \times v$: consequently $M : m :: v : V$; that is, v will be greater in the same proportion as m is smaller.

42. We wish it to be observed that we have two kinds of measure of the mass or quantity of matter in a body; the one is called the Statical, and the other the Dynamical mode of measuring it. It being known (as we have shown in § 39) that the earth's attraction is exerted with equal intensity upon every particle of matter, having no preference for one kind of matter over another, the quantity of matter contained in any body may be ascertained by finding a known quantity of matter which will just balance it, that is to say, by weighing it, or comparing it with a known weight, the standard of which is arbitrarily selected. (Thus, in this country, the ounce avoirdupois is so taken that 1000 of them will just balance a cubic foot of distilled water.) The Dynamical measure of a mass is found by ascertaining the velocity which a given pressure or impulse will impress upon it, the mass being inversely proportional to this velocity.

43. The following example may help to make the principles just laid down more clearly understood. *Ex.*—A body weighing 24 lbs. is moved by a constant force, which generates in it in one second a velocity of 3 feet per second; what weight would that force be sufficient to sustain?

To sustain a weight, a force must be exerted capable of producing a momentum equal to that which the force of gravity would produce in it, or the product of its mass (measured by its weight) and a velocity of 32·2 feet per second. The force therefore in the question is sufficient to produce a velocity of 3 feet

per second in a weight of 24 lbs., that is, a momentum of 3×24, and a velocity of 32·2 feet per second in a weight of x lbs., or a momentum of $32{\cdot}2 \times x$, where x is the quantity sought. We have therefore

$$32{\cdot}2 \times x = 3 \times 24, \text{ or } x = \frac{72}{32{\cdot}2} = 2{\cdot}236 \text{ lbs. nearly.}$$

An *impulsive* force capable of generating a uniform velocity of 3 feet per second would be able to sustain the same weight (2·236 lbs.) for one second of time; but it would require twice as powerful a force to enable it to resist the action of gravity for two seconds, three times for three seconds, and so on; since the velocity which gravity generates increases, its force being constant, exactly in proportion to the time.

44. Before we conclude we will say a few words concerning the law of gravitation. Hitherto we have treated it as if it were a force of always the same amount and acting in parallel lines, or in one constant direction, which we call downwards. And although (as we have said in Statics, § 14) its direction is towards the centre of the earth, and therefore it cannot act strictly in parallel lines; and though also, as we are now about to state, it varies in intensity when the distance of the body upon which it is exerted from the centre of the earth is altered, yet in nearly all the operations performed by man upon the earth's surface, the distance between two different points is so small in comparison with the earth's radius that lines drawn from them

to the earth's centre may, without sensible error, be regarded as parallel, and the force of gravity exerted upon them as being the same.

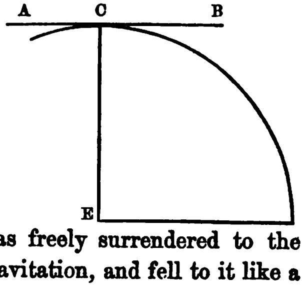
45. The actual law of gravity is this: It is a property of all matter by which every particle of matter in the universe is attracted towards every other particle with a force varying directly as the mass of the attracting body, and inversely as the square of the distance between the attracting and attracted bodies. So that if A and B are two bodies, of which A has twice as great a mass as B, A will attract B twice as much as B does A; and if they be removed to half the distance, the attraction of each upon the other will be four times as great as it was before. Mathematicians have proved that the attraction of spheres is the same as if all the matter contained in them were collected at the centre; the earth, therefore, being very nearly a sphere, its gravitation may be considered as a force tending to its centre.

46. By this law of gravitation, combined with the three laws of motion we have before enunciated, Sir Isaac Newton and his successors, following in his steps, have explained the motions of the great bodies of the universe (at least, those with the motions of which we have any acquaintance) in a way that may now be looked upon as almost complete. And this, the doctrine of physical astronomy, founded as it is upon the most profound mathematical reasoning combined with accurate and refined observations carried on through a long series of years by means of instru-

ments which have been the product of the greatest human skill and ingenuity, may well be considered as the most magnificent result in the field of natural knowledge that man has achieved.

47. The first step in the chain of reasoning by which Newton was led to this grand generalization and to extend the force of gravity from the earth to the heavens, from its long-known effects upon bodies on the earth's surface to similar effects upon the celestial spheres, may perhaps be profitably indicated here; and so we will conclude this short sketch of the first principles of Dynamics.

48. The Moon being conceived to have a projectile force impressed upon it, by which it is moving with a constant velocity in the direction AB of a tangent to its orbit round the Earth, the centre of which is E, and the Earth's attraction acting upon it in the direction CE; by the second law of motion, the latter force draws the Moon just as far from the tangent to her orbit as if the projectile force had no existence;



i. e. as if the Moon was freely surrendered to the action of the Earth's gravitation, and fell to it like a body on the Earth's surface—with this difference, that the force of gravity being less than at that surface on account of the greater distance from the centre, the Moon will be drawn through a much smaller space in the same time. Now the Moon is

distant about 30 diameters of the Earth from the Earth's centre, and therefore she is 60 times as far from the Earth's centre as is a body at the Earth's surface; consequently as gravity varies inversely as the square of the distance, the Earth must, if our principles are correct, exert an attractive force upon the Moon 60×60, or 3600 times less than it does upon a body at its own surface. And it has been found by observation that the Moon actually *is* drawn from the tangent to her orbit by that amount, *i. e.* about the 3600th part of 16·1 feet in one second of time.

Section III.

HYDROSTATICS AND HYDRODYNAMICS.

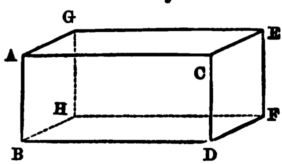
1. As Statics and Dynamics investigate the laws of the Equilibrium and Motion of bodies in general, Hydrostatics and Hydrodynamics are concerned with the discussion of those which are peculiar to Fluids when in a state of rest and of motion.

2. A fluid differs from a solid body in this respect, that its molecules or separate particles possess so great a mobility or capability of motion amongst each other that the application of the slightest force to any one of them is sufficient to displace it from its position relative to the rest.

3. Fluids are divided into elastic and non-elastic. Elasticity is a property of certain bodies by which they admit of compression by pressure into a smaller space than that which they actually occupy, and, when the pressure is removed, expand again, so as to re-fill the whole space they occupied before the pressure was applied. So far elasticity is a property possessed by some solids as well as fluids; but the peculiarity of elastic fluids is that, when unrestrained by external pressure, they expand almost indefinitely, so that if a very small quantity is introduced into a vessel however large, it is almost immediately filled by it, or at least the molecules of the fluid (owing apparently to their repulsion for each other) are dif-

fused through the whole space contained by the vessel. It is usual to treat of elastic fluids under the separate head of Pneumatics; we shall therefore in this section speak chiefly of non-elastic fluids (of which the most important, because the most universally met with, is water), reserving the consideration of the points in which the elastic fluids differ from them for the next.

4. The great characteristic of fluids is the equality with which any pressure exerted upon them is transmitted through them in all directions. To make the meaning of this principle clear, we will suppose a solid and a fluid body to be unacted upon by the force of gravitation, and, being at rest, to be free to move in any direction in which they are urged by external pressure. The solid, on account of the cohesion of its particles, will, when pressure is applied, move, in one mass, in the direction in which the pressure acts, or towards the opposite side to that at which the pressure is applied. But the fluid, by reason of the mobility of its particles, will suffer those particles to be urged in different directions, transmitting the pressure unchanged in intensity through the whole mass of fluid in every direction. Thus, let ABCDEFGH be a rectangular vessel containing such a mass of fluid. If a pressure equal to 100 lbs. be ap-

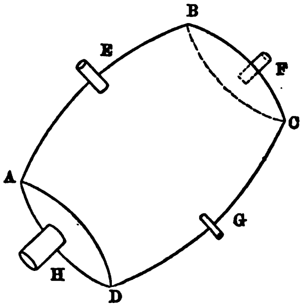


plied to the whole side CDEF, and if that side be supposed to contain 100 square inches of surface,

this will be equivalent to a pressure of one pound upon every square inch on that side. This amount of pressure then will be transmitted through the whole mass of fluid contained in the vessel in every direction; so that each of the remaining five sides will be subjected to a pressure of one pound upon every square inch of the surface from within. The whole mass therefore can only be kept in a state of equilibrium by applying an external pressure on each side proportional to the area of surface which it contains, so that every square inch of the whole surface on every side may undergo an exactly equal pressure.

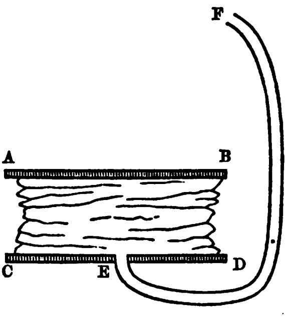
5. In the experiments we are able to make, fluids are under the influence of the law of gravity, to which, equally with solids, they are obedient. Now the force of gravity, acting constantly in one direction or downwards, causes a pressure through the whole mass of fluid, which is transmitted, like other pressures, equally in all directions. But let a cylindrical vessel be filled with fluid;—as each layer of fluid has to undergo the pressure of all the superincumbent layers, the pressure upon each increases with the depth; and if the fluid be of equal density throughout, the amount of pressure upon a square inch of fluid so increases as to be exactly proportional to the weight of the superincumbent mass of fluid, and therefore to the depth below the surface. (Indeed we shall see presently (§ 11) that, whatever be the shape of the vessel, the pressure upon a given area is always exactly proportional to the depth below the surface.) Consequently the pressure pro-

duced by the action of gravity, combined with the law of equal transmission of fluid pressure upon the interior sides of a vessel filled with fluid, is less upon the upright sides in proportion to their area than upon the bottom of the vessel; and less upon the upper than upon the lower parts of the upright sides. But if these pressures be sustained or counteracted by equal pressures from without, any additional pressures, that equilibrium may be maintained, must be in the proportion stated in the last article. So that if ABCD be a vessel of any shape and in any position filled with a fluid of equal density throughout, and if there be orifices of different sizes and at different parts of the vessel, into which fluid-tight pistons are inserted; then, that equilibrium may be maintained, or the pressure of the fluid from within the vessel resisted, an external pressure must be applied upon each piston proportional to the area of surface into which it is fitted, together with so much additional pressure as may be necessary to resist the force of gravity of the fluid, which is transmitted through it so as to be jointly proportional to the area of surface pressed and the depth of that surface below the highest part of the fluid. And, by taking this into account, the law of



transmission of fluid pressure is capable of being experimentally verified.

6. A very curious result is deducible from this principle of equal transmission. Since any pressure applied is transmitted proportionally to the magnitude of surface pressed, the whole pressure produced upon any surface may be increased to any extent by increasing that surface. This may be illustrated by the following remarkable experiment. Let ABCD be a vessel consisting of two boards connected together by leathern sides, the tightening or extension of which raises the upper board, which is moveable, further above the lower, which is supported. Into an opening through the lower board CD, a small tube EF is introduced, which, as well as the vessel itself, is filled with water. Now by making the board AB sufficiently large, it is found that a man standing upon it and blowing into the tube at F is able to lift his own weight without difficulty; the force thus



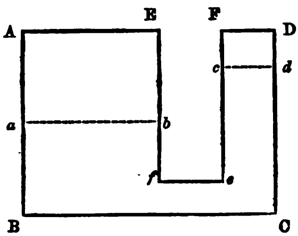
exerted upon the water in the tube being increased upon the whole board AB in the proportion of the number of times which its area contains that of the transverse section of the tube. This great multiplication of a small force by the transmission of fluid pressure has been called the Hydrostatic Paradox.

7. We have shown (§5) that the amount of pressure to which any part of a mass of fluid is subjected in consequence of the force of gravity transmitted through it is proportional to the depth of the part below the upper surface, or highest part of the fluid. From this it follows that the upper surface of a fluid at rest under the action of gravity alone is a horizontal plane, since otherwise, if a part of the surface were higher than the rest, those parts of the fluid which were under it would exert a greater pressure upon the surrounding parts than they received from them, so that motion would take place amongst the particles and continue until there were none at a higher level than the rest, *i. e.* until the upper surface of the whole mass of fluid became a horizontal plane.

8. Since fluids transmit pressure equally in all directions, when a mass of fluid is at rest or in a position of equilibrium, every particle of that mass must exert in every direction a pressure equal to that to which it is itself subjected. For, to create equilibrium, every force or pressure must be neutralized by an equal opposing force or pressure.

9. So that when a mass of fluid is contained in several vessels possessed of free communication with each other, the same conditions of equilibrium must subsist as if it were contained in one, and the fluid when at rest must have its upper surface at the same level in all the vessels. For let ABCD be a reservoir consisting of two cylinders or vessels of either equal or unequal sizes. Then if water is poured into them and is at unequal heights, as *ab*, *cd*, in the

two cylinders, any particle under *cd*, as at *e* for instance, will undergo a pressure arising from the weight of the column of fluid *ce* above it, which it will transmit unchanged in intensity, first laterally or in the direction *ef*, and then upwards, or in the direction *fb*. To

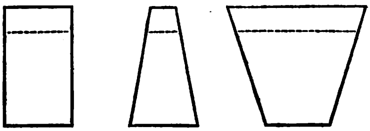


this pressure a particle of fluid at *f* has only to oppose that arising from the weight of the column of fluid *bf* above it, which is of course smaller if the height *bf* is less than *ce*. Motion therefore must arise from this unequal pressure, and must for the same reason continue until the fluid stands at the same height in the two cylinders, when equilibrium will ensue.

10. The same law will obviously hold good when the different vessels are of different shapes and placed in different positions. For by the way in which fluid pressure is transmitted, the pressure which any particle undergoes is proportional to its depth below the highest part of the fluid with which it has free communication, whether that part is actually in a vertical line with it and connected by successively superposed particles or not.

11. The pressure therefore exerted by a mass of fluid upon the bottom of a vessel containing it is proportional to the area of the base, and the height of the upper surface of the fluid above the base, and is wholly independent of the shape of the vessel and

of the quantity of fluid contained by it. So that if we have three vessels, such as those represented in the figure, whose bases are equal, but the first is cylindrical or of equal width throughout, the second tapers towards a point at the top, and the third in-



creases in width towards the top; and if fluid (always supposing it to be either the same kind of fluid or at least of equal density) be poured into each till it stands at the same height in all, the pressure exerted upon the base of each vessel will be precisely the same—a result capable of being verified by experiment.

12. A remarkable consequence may be deduced from this, and may also be experimentally shown to be true. If a vessel of water have introduced into it a long vertical pipe, which is also filled with water, then, if the pipe be sufficiently long, no vessel will be strong enough to resist the amount of the fluid pressure to which this arrangement subjects it. So that if the pipe be sufficiently thin, a few ounces of water will burst the strongest vessel ever made—the pressure upon the bottom being magnified in proportion to the number of times its area contains that of the transverse section of the pipe.

13. From the above principles it will be easy to estimate the amount of pressure exerted by a mass of fluid upon any part of a vessel filled with it, whose sides are rectangular and vertical. For the lateral pressure being the same as the vertical pressure at the same depth, the whole lateral pressure upon any side will be the mean of all the pressures at every depth from the highest to the lowest part of that side, or will be the same as if the pressure were everywhere equal to what it is at a point midway between the bottom and the top of the vessel. The pressure therefore upon a vertical side will be the half of what it would be if that side were horizontally immersed in the same fluid at the depth at which its base actually is—that is, half the weight of a mass of the fluid whose base was equal in area to that side and its height equal to the actual height of the fluid in the vessel.

14. If, for example, a cubical vessel is full of water and each side of the cube a foot in length, so that the area of each side of the vessel is one square foot and the content of the vessel one cubic foot, the pressure of the water upon the base will be the whole weight of the water, or 1000 ounces avoirdupois, while that upon each of the four vertical sides will be (since their area is the same as that of the base) half the same weight, or 500 ounces = $31\frac{1}{4}$ lbs.

15. We have now to consider what takes place when a solid body is wholly or partially immersed in a fluid. Now if we conceive any part of a mass of fluid to become solid, the only difference between its

action and that of the fluid which previously occupied its place, will be that it will no longer exert any lateral pressure; but the downward pressure or weight will not be different from what it was before. If, therefore, this solid be of the same density or of the same specific gravity (so the comparative weight is called) as the fluid, it will rest in equilibrium wherever it is placed; if it be heavier, it will sink, the opposing upward fluid pressure being unable to sustain it; if lighter, it will rise, as that pressure exceeds its weight.

16. In the case of the body heavier than the fluid in which it is immersed, there is required to sustain it an upward pressure equal to its own downward pressure or weight: the fluid only exerts a pressure equal to the weight of a mass of itself of the same bulk as the solid. From this it follows that if a body be so suspended and weighed, its apparent weight will be less than its actual weight by that of a mass of the fluid of equal bulk, since the latter weight helps to sustain it. It is obvious that this affords a ready means of comparing the weights of a solid body and of an equal bulk of a fluid in which it is immersed—that is, of ascertaining the specific gravity of the solid as compared with that fluid.

17. It is of course necessary to select some standard of specific gravity, to which to refer all others. And the one chosen is the weight of a cubic foot of distilled water, which is 1000 ounces avoirdupois (see Dynamics, § 42). So that, if a cubic foot of any substance weighs twice this quantity, its specific

gravity is said to be 2; if only half as much, ½ or 0·5, &c.

18. It will be convenient to illustrate by an example the method just explained of finding the specific gravity of a solid body by immersing it in fluid.

Ex.—A body weighs 6 lbs., but when suspended in water appears to weigh only 4 lbs.; what is its specific gravity?

Here the loss of weight in water is 2 lbs.; so that a quantity of water of the same bulk as the body weighs 2 lbs., while the body itself weighs 6 lbs. The specific gravity, therefore, of the body is to that of water as 6 to 2, or as 3 to 1. That of water then being considered to be the unit of reference, that of the solid suspended in it is 3.

19. If the same solid be weighed in two different fluids, the above principle enables us to find the specific gravity of one of the fluids as compared with that of the other. Thus, if a body of the real weight of 10 lbs. appears when suspended in water to weigh 8 lbs., and when suspended in some other fluid 9 lbs., *i. e.* loses 2 lbs. of weight in the former case and only 1 in the latter, it will follow that a quantity of water of the same bulk as the solid weighs 2 lbs., and the same quantity of the other fluid only 1 lb.; the specific gravity, therefore, of the fluid in question is but half that of water, or is equal (water being the standard) to 0·5.

20. When a solid body is lighter or of less specific gravity than a fluid in which it is placed, the upward pressure of the fluid will be greater than the weight

or downward pressure of the solid; the latter will consequently only partially sink, that is, it will *float* in the fluid. It will, in fact, sink until it has displaced a mass of fluid of the same weight as itself; for the downward pressure of the solid is equal to the whole of its weight, which acts as if it were all collected at its centre of gravity upon the part of the fluid with which it is in contact; and the upward pressure of the fluid must also, when equilibrium is established, be just equal to this. Now this upward pressure must be exactly equal to the weight of that portion of the fluid which was displaced to make room for the part of the solid sunk below the surface of the fluid; since it was just sufficient to counteract that weight before the immersion of the solid, when the fluid preserved an unbroken level surface. Consequently the weight of the whole solid must be the same as that of the part of the fluid which is displaced to allow the solid to sink until equilibrium is restored. That is to say, a solid placed in a fluid of greater specific gravity than itself will sink until the portion of the fluid displaced by it, which is of course equal in bulk to the part of the solid below the surface of the fluid, is of the same weight as the whole solid. For instance, if a solid body of three-quarters the specific gravity of water be placed in a vessel of water, it will sink until three-quarters of its bulk is below the surface—that quantity of water which is displaced by it being just equal in weight to the whole solid.

21. So far we have spoken of the equilibrium of

fluids when undisturbed by the presence of anything else, and also when solid bodies of specific gravities differing from those of the fluids are completely or partially immersed in them. This completes what we have to offer on the subject of Hydrostatics.

22. We will now proceed to say a few words on Hydrodynamics or Hydraulics (as it is sometimes called), which treats of the *motion* of fluid bodies.

23. If an aperture be made in the bottom of a vessel containing fluid, the fluid will flow out in obedience to the law of gravitation, both by the direct action of gravity upon the lowermost particles, and by its indirect action causing the particles above to exert a greater or less pressure upon those lowermost particles, according to their depth below the upper surface of the fluid. If the aperture be very small, so that its size may be neglected, it makes no difference whether its direction be sideways or downwards, or, by means of a pipe inserted, turned even upwards; for, in consequence of the equal transmission of fluid pressure, so long as the depth is the same, the force exerted upon the particles will be also precisely the same.

24. The law expressing the relation between the depth of any part of a mass of fluid, and the velocity with which it will issue through an aperture in a vessel containing it, was discovered experimentally by Torricelli, and is therefore called Torricelli's theorem. It is this:—If a small aperture be made in a vessel containing fluid, the velocity with which the particles of fluid will issue through it will be the

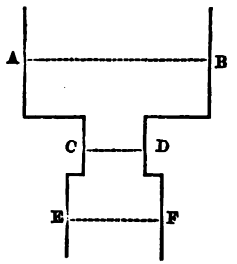
same as if they had fallen freely under the action of gravity through a height equal to the depth of the aperture below the upper surface of the fluid. So that if a pipe be inserted into the aperture and bent upwards, the velocity of the issuing fluid will be just sufficient to project it to the height of the upper surface of the fluid in the vessel—neglecting of course the effects of friction and of the resistance of the air. It matters not what is the density or specific gravity of the fluid; if the depth be the same, the velocity will be the same; for although a heavier fluid exerts a greater pressure upon its lower particles, yet they present a proportionally greater mass to be moved, so that their resulting velocity will be precisely the same.

25. In Dynamics (§ 16) we showed that when a body falls freely under the action of gravity, if its velocity at any time be called v, the space through which it has fallen to generate that velocity s, and the force of gravity g, then we have the equation $v=\sqrt{2gs}$. Hence, if s be now made to represent the depth of an orifice in a vessel containing fluid below the surface of that fluid, the same equation will give the velocity with which the fluid will issue through that orifice. This velocity, therefore, and consequently the quantity of fluid which flows through in a given time, will be directly proportional to the square root of the depth of the orifice.

26. And as we showed in Dynamics (§ 18) that the space described by a body falling from rest in any given time was equal to one-half that which it

would have been had it fallen during the whole time with the velocity which it had at the end of it, so now it might be shown that if two vessels be filled with fluid, and small equal orifices be made at the same depth below the surface in each, but whilst the one is allowed to empty itself, the other is kept full by pouring in fresh fluid, the quantity of fluid discharged by the vessel kept constantly full in the time the other takes to become empty, will be double the whole quantity thus discharged by the other. For the velocity in the former case will continue uniform, whilst that in the latter, at first the same as the other, will diminish to nothing (in consequence of the continuous decrease in the depth of the issuing fluid below the surface) by identically the same stages reversed as a body falling freely under the action of gravity would acquire that velocity.

27. Before concluding the subject of the motion of fluids, we will show how the velocity of a fluid flowing through a pipe will vary when the bore of that pipe varies in size. Let a fluid be made to flow through the tube ABCDEF, which is placed, either vertically or otherwise, in such a way that the whole tube, which is of several different widths, is kept constantly full. Since the pipe is always full, the quantity of fluid which passes during any given time through any plane AB must be precisely equal to



that which passes in the same time through the planes CD or EF, which are of smaller size than AB. Now if the same quantity passes in the same time through two areas differing in size, it is evident that the velocity must be greater in passing through the plane of small area than in passing through that of large area; that it must, in fact, be inversely proportional to the size of the area. Hence the velocity of a fluid in passing through a pipe the bore of which varies in size in different parts of the pipe, is inversely proportional to the area of the bore or transverse section of the pipe at that part.

Section IV.

PNEUMATICS.

1. Pneumatics is concerned with the discussion of those properties of elastic fluids which are peculiar to them as such. Elastic fluids, like liquids or non-elastic fluids, are subject to the law of gravity; and like them also they transmit any pressure exerted upon any of their particles equally in all directions. But whereas, in liquids, the pressure exerted is entirely due either to their own weight or to the application of some external pressure, transmitted in this way, the pressure of an elastic fluid depends chiefly upon its elastic force.

2. It has been already stated in the preceding section (Hydrostatics, § 3) that an elastic fluid differs from a non-elastic in this respect, that if pressure be exerted upon the former, it yields to it by being compressed into a smaller bulk; and, on the contrary, if pressure be removed, expands again so as to diffuse itself equally through any space, however large, which it is allowed to enter. Space, therefore, cannot be said to be filled by an elastic fluid in the way in which it would be filled by one which was non-elastic. Two or more elastic fluids may be brought into the same space; and if this be done, both will quickly diffuse themselves through the whole space, so that every part of it will contain an exactly equal proportion of the two fluids.

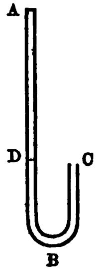
3. Elastic fluids are divided into two classes. Those which are not seen (at least under ordinary pressures and temperatures) in any other form are called gases; those which, when the temperature to which they are exposed is sufficiently lowered, assume the form of non-elastic fluids, are called vapours. Steam, or vapour of water, affords a common example of the latter. But one of the former, atmospheric air, so necessary to us in every respect, and with which accordingly we are always surrounded, will occupy the principal part of our attention here.

4. Atmospheric air consists indeed not of one gas, but of a mixture of two gases in different proportions. But in consequence of the property of elastic fluids, which we have already mentioned (§ 2), the two, which are called oxygen and nitrogen, are so completely mixed and equably distributed through any space containing them, that the air may be conceived to be but one mass, every part of which is composed of exactly the same proportion of its two component gases, and may therefore be treated, as to its mechanical properties, as if it consisted but of one kind of gas.

5. The weight of the masses of air which usually come under our notice is so small that its effect may be entirely neglected in comparison with that produced by the elastic force. But the effect of gravity upon the whole mass of air contained in the earth's atmosphere produces a pressure at the earth's surface so great in amount that it can never be neglected, and is indeed of the utmost importance in many

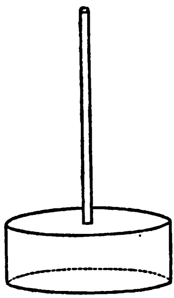
points of view. Before we proceed any further, we must describe the means by which that pressure is measured.

6. Let ABC be a bent glass tube filled with a fluid, closed at A, but open at C. Now, from the principles laid down in Hydrostatics, if there were no pressure exerted by the atmosphere, or if the instrument were placed in a vacuum, the fluid would sink in the long end AB of the tube until it stood no higher there than in the short end; so that if the tube were originally quite full of fluid, it would flow out at the opening at C, until it stood at the same level in the tube AB as that opening (*i. e.* at D). The same would be the case if both ends of the tube were open, so that the fluid in both had to sustain the same pressure from the atmosphere. But by the arrangement indicated, the tube being previously filled with fluid, and then placed in the position shown in the figure, the pressure of the external air is exerted at the opening C upon the fluid contained in that end of the tube only. The fluid therefore will sink at the end A, and flow out at the opening C only until the fluid pressure in the long end of the tube is exactly equal to that in the short end, together with that of the whole atmosphere acting at C.



7. If the tube were straight instead of bent, and after being filled with fluid, inverted with the closed end upwards, and the open end immersed in an open trough or vessel full of fluid, a similar effect would

follow. For the pressure of the atmosphere acting upon the fluid in the open vessel, and being transmitted equally in all directions, with a force proportional to the area of surface pressed, the fluid in the tube would sink until a column was left of such a height that its pressure upon the area contained by its base should exactly equal that exerted by the atmosphere upon the same area. Above the column of fluid thus remaining in the tube, a vacuum would of course be left: this has obtained the name of the Torricellian vacuum, from the experiment having been first made by Torricelli in the year 1643.



8. This, then, is the principle of the barometer,—the instrument for measuring the pressure or weight of the atmosphere. The fluid usually employed (being for various reasons, particularly its great specific gravity, the best) is quicksilver; and it is found that the pressure exerted upon any area of surface by the whole atmosphere, is equal to that of a column of mercury of about 29 or 30 inches in height. (We say 29 or 30, because it admits of changes from time to time, owing to various causes which tend to increase or diminish the pressure of the atmosphere.) If water is employed, it is found that the atmosphere is capable of sustaining the pressure of a column about 34 feet in height. This pressure, exerted by the atmosphere,

is equal to nearly 15 lbs. upon every square inch of surface.

9. When we ascend an elevation, there being less atmosphere above us, the pressure arising from it is also of course less, and accordingly it is found that the height of the mercury in the barometer is less than it is below. The air being an elastic fluid, and consequently yielding to pressure by compression, and the pressure of the superincumbent mass being greater at the earth's surface than at a position elevated above it, it follows that the air at the surface is denser than that above it, and considerably so than that at great elevations. The height therefore of the mercury in the barometer (usually called the reading of the barometer) does not diminish equably as we ascend above the earth's surface, as it would do if the air were, like a mass of non-elastic fluid, of uniform density throughout. In fact, at an elevation of about 7000 feet, or little more than a mile, we find that the barometer reads, instead of 30 inches (the ordinary height near the level of the sea), only about 23, proving that we have there ascended through nearly one-quarter of the whole atmosphere, and have only three-quarters of its weight still above us. But to ascend through another quarter, where the barometer indicates a pressure equal to that of only 15 inches of mercury, it is necessary to rise to an elevation of 18,000 feet, or nearly 3½ miles, which is considerably more than double the former.

10. When the law of rate of decrease of pressure for increase of elevation has been ascertained, the

barometer may be used as a means of measuring the height of the observer above the earth's surface. In this way heights attained in balloon ascents are known. Thus, in the very remarkable ascent of Mr. Glaisher on the 5th of September 1862, for the purpose of determining the thermometric and hygrometric states of the air at great elevations, the barometer was observed by him to register less than 10 inches, indicating that he had then* attained a height of nearly six miles.

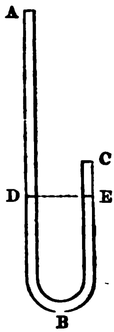
11. If the atmosphere were of the same density throughout as it is at the sea-level, its pressure, known (as we have just shown) by the barometer, would lead us to the conclusion that it extended only about five miles in height. Its actual height, however, is not less than forty-five miles, as is known by its perceptible effects in producing refraction in the rays of light passing through it—alone sufficient to prove how much rarer it must be above than at the earth's surface.

12. The elastic force of the air is dependent not only upon the pressure to which it is subjected, but also upon the temperature or amount of heat, any change in which produces a corresponding change in the elastic force. If the temperature remains the same, the elastic force is such that the space through which a mass of air is diffused will vary inversely as the pressure to which it is exposed; so that the density of the compressed air will be directly proportional to

* We say *then*, because he ascended even higher, but was unable to continue his observations.

the pressure which compresses it. This, which is called Boyle's or Mariotte's Law, from its discoverers, is experimentally proved to be true as follows.

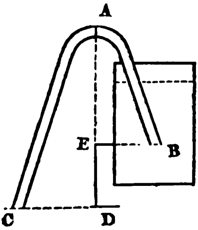
13. Let there be a glass tube, open at both ends, of the form represented by ABC in the figure. Let a small quantity of mercury be poured into either leg, and it will of course, by the law of hydrostatic pressure, rise to the same height (DE for instance) in both. Now let the end C of the shorter leg be closed. The position of the mercury will be found not to have changed; for, although the pressure of the atmosphere above C is cut off, yet the air in the space CE being of the same density as it was before, its elastic force will not be altered, which will enable it to sustain the pressure of the atmosphere acting upon the mercury in the other leg of the tube. Now let more mercury be poured in at A; it can only rise in the shorter leg by compressing the air contained in the space EC; and it is found that to compress it into half the space it occupied before, such a quantity of mercury must be poured into the leg AB as suffices to make its height in that leg exceed that in the other by about 30 inches, that quantity, as already stated, exerting a pressure equal to that of the whole atmosphere. The conclusion to be drawn is, that a pressure equivalent to that of two atmospheres compresses a mass of air upon which it is exerted into half the space which that of one atmosphere does, and of course doubles its density.



And, similarly, it is found that a pressure the same as that of three or four atmospheres trebles or quadruples the density of air previously subjected to that of one only. We infer therefore that the space occupied by an elastic fluid varies inversely as the pressure by which it is compressed—provided, as already mentioned, the temperature remains the same; for any increase of this increases the force by which elastic fluids tend to expand and diffuse themselves through a larger space.

14. We will now proceed to describe a few instruments the action of which depends upon atmospheric pressure, and by which the effects of that pressure may be illustrated.

15. First, then, of the Siphon. If a bent glass tube BAC be filled with water by closing the end of the shorter leg, and pouring the fluid into the longer, and then, after inverting the tube as shown in the figure, the shorter leg be immersed in a vessel of water, and both ends opened, the water will

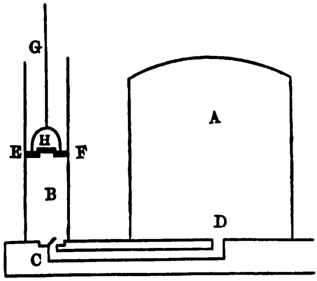


flow out at the long end, for this reason: the pressure of the atmosphere acts equally upon both ends, tending to sustain the water; but that pressure is opposed by the downward pressure arising from the weight of the water, which is greater in the longer leg than in the shorter. The force therefore urging the particles of the fluid at A towards B will be equal to the pressure of the atmosphere diminished by that due to the

weight of a column of water of the vertical height of A above C (=AD); the force, on the other hand, urging those particles towards C will be equal to the atmospheric pressure diminished by that due to the weight of a column of water of the height of A above B (=AE). The former force being smaller than the latter, and the two being opposed to each other, the particles will yield to the influence of the latter, and will flow towards C; others will take their place and will follow them, so that there will result a continual motion of the water from the vessel, through the tube, finally flowing out at the end C; and this will continue as long as the end B is below the surface of the water in the vessel. Of course, if the height AE is greater than 33 feet, the weight of the water in the short leg being more than sufficient to counteract the pressure of the atmosphere, the instrument will not act, but the water will flow out at both ends until the tube is empty. If the siphon be not originally filled with water, but the air be partially withdrawn from it by suction, the water may be drawn from a vessel in which the short end is immersed in the same way as above, simply from the effect of the uncompensated pressure of the atmosphere upon the water contained in the vessel.

16. The next instrument we shall describe is the Air-Pump, the use of which is to exhaust the air from a closed vessel called a receiver. There are various modifications of it, but we shall content ourselves with explaining it in its most simple form. In the annexed figure, A represents the receiver,

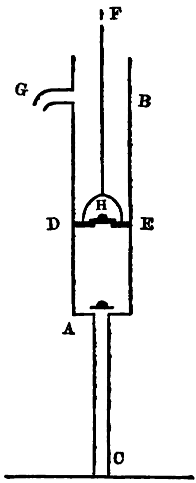
from which the air is to be exhausted; B a cylinder communicating with the receiver by the tube CD; in it a piston EF (closely fitted to it, so that no air can pass between) is moved up and down by the rod G; the piston is provided with a valve H opening upwards; and there is a similar valve at the end C of the tube CD. Now let the receiver be full of air at the ordinary atmospheric pressure, and the piston EF be at the



bottom of the cylinder, or nearly so. Then let the piston be drawn up; as it rises, the valve H will remain shut, in consequence of the pressure of the external atmosphere. The air therefore in the part of the cylinder below the piston will diffuse itself over a larger space (left by the gradual rise of the piston), and will be unable to resist the pressure produced by the superior elastic force of the air in the receiver and tube, so that the effect of the latter will be to open the valve at C, part of the air passing from the receiver into the cylinder until that contained in both is of the same density. Now let the piston be pushed down again; the valve at C will shut, and the air in the part of the cylinder below the piston, being compressed by the descent of the piston, will overcome by its pressure that of the

external atmosphere, so as to open the valve at H and escape, while the air in the receiver remains undisturbed. It is evident therefore that the effect of the stroke has been to take away part of the air from the receiver, that which is left being less dense than it was before; and by a repetition of strokes of the piston, this exhaustion will be continued until the quantity of air left in the receiver is very small indeed, though it cannot be reduced absolutely to nothing.

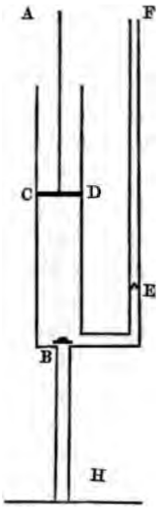
17. We will now explain the action of the common or sucking-pump, used for drawing up water from a well or cistern. It is very similar to that of the air-pump, just described. Let AB in the figure represent a cylinder, at the lower part of which is a pipe AC partly immersed in the water of the cistern, some of which it is desired to raise. A piston DE is worked up and down by means of the rod F; there is a spout at G, and valves in the pipe and piston at A and H, both opening upwards. Now let the piston DE be at the bottom of the cylinder, or nearly so, and let it be raised upwards. The valve at H will remain shut, and the air in the lower part of the cylinder below the piston, being diffused over a larger space, will be unable to resist the pressure produced by the



superior elastic force of the air in the pipe, which will therefore open the valve at A, part of it passing into the cylinder. Then the rarefied air in the pipe will be insufficient to counteract the pressure of the external atmosphere upon the water in the cistern, the consequence of which will be that part of the water will be driven up the pipe, until the pressure of the rarefied air and the water thus ascended into the pipe are jointly equal to that of the atmosphere. So long as the piston continues to rise, the air in the cylinder below it will be more and more rarefied, and the water will continue to rise in the pipe. When the piston descends again, the valve at A will shut, and the condensation of the air in the lower part of the cylinder will cause it to exert a pressure which will open the valve in the piston, and allow part of the air to escape. A repetition of these strokes of the piston will, as in the air-pump, more and more rarefy the air in the pipe and in the part of the cylinder below the piston, causing it to be less and less able to resist the pressure of the external atmosphere, so that the water will continue to rise higher and higher in the pipe, until it enters the cylinder through the valve A. On the next descent of the piston, the water will force its way through the valve it contains at H, will be lifted up by the piston when it reascends, until it flows out through the spout G. It is evident from this explanation that the pump will not act if the valve at A be more than 33 feet above the surface of the water, since the pressure of the atmosphere is

not sufficient to sustain a column of water of greater height than this. If the spout also be less than 33 feet above the surface of the water, the water will gradually fill the cylinder as well as the pipe, and the flow from the spout will in that case be continuous.

18. There is a modification of the pump called the Forcing Pump, by means of which water may be forced to a greater height than it can be raised by the common pump. In this instrument, the piston CD contains no valve, but is quite solid, as well as air-tight; and at the bottom of the cylinder AB there enters a pipe EF of any length we please, provided with a valve E at its lower part (or near the place where it communicates with the cylinder) opening upwards. The water is raised by the rarefaction of the air in the pipe BH (which here also must not exceed 33 feet in length) into the cylinder, as in the sucking-pump. The descent of the piston afterwards forces it through the valve E into the pipe EF, the valve subsequently closing so as to prevent its return.



Section V.

OPTICS.

1. We shall not here speak of the theories of light, but shall commence by defining it to be that substance or action upon a substance by means of which objects are rendered visible, or are made to produce such an effect upon our eyes as constitutes the sense of sight.

2. Now this substance or action, whichever it be, in the first instance always emanates from, or originates in, a self-luminous body, which is called a source of light. And the first and most obvious facts about it are that this emanation takes place in straight lines and in all directions, and is transmitted through space at an immense velocity: if the emanation took place in a vacuum, and the light encountered no obstacle in its course, these are indeed the only facts we could ever have known about it. The science of Optics, then, is principally occupied in inquiring how light acts when it passes through, or meets with, objects of different form and substance.

3. We must premise the following definitions:—

A *ray* of light is the smallest quantity of light which can proceed in any direction; it is treated in all reasonings as a mathematical line, that is to say, as possessing only length, but neither breadth nor thickness.

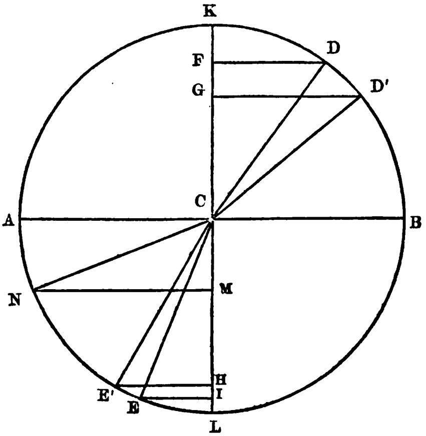
A *pencil of rays* is a small bundle of rays proceeding originally from a luminous point. If the point is near, the rays will be divergent; if it is at a great distance, they will be sensibly parallel; and their direction may be so changed that they may become even convergent.

A *transparent* body or substance is one which permits light to pass through it; an *opaque* body is one which does not.

4. When rays of light are incident upon an opaque substance, or such a one as prevents their passing through it, they are reflected or thrown back in such a direction as is expressed by the following law:— If a perpendicular be drawn to the plane on which a ray is incident, at the point of incidence, the reflected ray will be in the plane formed by that perpendicular and the incident ray; and the incident and reflected rays will form equal angles with the perpendicular on opposite sides of it.

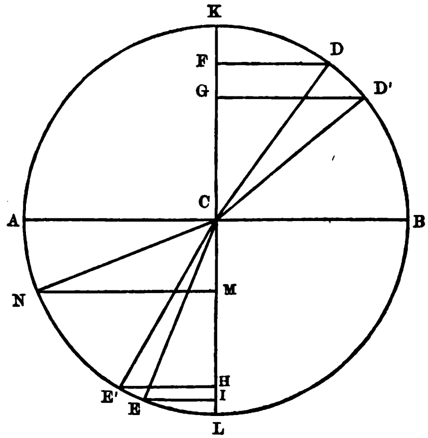
5. When a ray of light falls upon a transparent body or medium (as such a body is then called) it passes through the medium, but with a direction not the same as that at which it entered it, but making a certain angle with that direction, differing according to the nature of the medium. This change of direction is called Refraction. If the ray passes from a rarer into a denser medium, it is refracted more nearly *towards* the perpendicular to the surface of the medium; if from a denser into a rarer medium, *further from* that perpendicular. As in the case of reflexion, the incident ray, the perpendicular to the

surface at the point of incidence, and the refracted ray will be all in the same plane. The discovery of the law expressing the amount of change of direction gave much trouble to investigators, but was at last accomplished by Snell about the year 1621. It is this:—the sine of the angle which the incident ray makes with the perpendicular to the surface at the point of incidence is to the sine of the angle which the refracted ray makes with that perpendicular in a constant ratio. As we wish to avoid using trigonometrical terms in this treatise, we will express this law by a geometrical construction. Let ACB be the surface of a medium (as water) of greater density than



another medium (as air) above it, DC, D′C two incident rays, and CE, CE′ the corresponding refracted rays. KL is the perpendicular to the surface at the

point of incidence; to it DF, D'G, EI, E'H are perpendicular. Then, whatever be the angle of incidence, we shall always have this proportion:— DF : EI : : D'G : E'H. As this then measures the refractive power of any substance, it is called the index of refraction for that substance; *i. e.* if the space



above AB be void of matter, and the space below be filled with any substance, the ratio of EI to DF (the former quantity, for convenience, being considered to be unity) is called the refractive index of that substance. We have said that if the light passes from a rarer into a denser medium (from air into water for example), the angle of refraction is less than the angle of incidence; or, in the above figure, the angles ECL, E'CL are respectively less than the angles DCK, D'CK. It is obvious that if the incident ray is

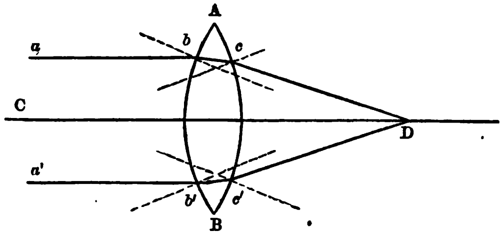
perpendicular to the surface, the direction will not be changed at all, or no refraction will take place; while the more it is inclined to the perpendicular to the surface, the more the direction will be changed, or the greater will be the amount of refraction. Another circumstance calls for attention here: if the light passes from a denser into a rarer medium, since DF, D'G, &c. are always larger than EI, E'H, &c., when the incident ray makes a very large angle with the perpendicular to the surface (such as NCL), the fourth proportional to EI, DF, NM, or to E'H, D'G, NM will be a larger quantity than BC, so that, according to the law, no refracted ray can exist. And indeed in this case the whole of the light is reflected within the medium and none refracted out of it. The smallest angle at which this takes place is called the angle of total reflexion, or sometimes the critical angle, of the medium.

6. We will now show how rays of light are refracted through a lens, preparatory to speaking of actual vision, or the means by which all objects are seen by us.

7. We must first define a lens, thus:—A lens is a refracting medium bounded by two spherical surfaces; the line joining the centres of the spherical surfaces is called the *axis* of the lens.

8. Now if either a parallel or a conical pencil of rays the axis of which is coincident with the axis of a lens, fall upon that lens, the direction of the separate rays will be altered in passing through the lens, and altered symmetrically. If, for example, the lens

be double convex (or convex on both sides) and the pencil of rays a parallel one, it is easy to trace the course of the separate rays by the law of refraction stated above. Let AB be the lens, CD its axis, and *ab*, *a'b'* parallel rays equidistant from the axis and on



either side of it, incident upon the lens at *b* and *b'*. The dotted lines representing the perpendiculars to the surfaces of the lens, at entering the lens the ray *ab* is refracted into a direction more nearly *towards* the perpendicular or normal to the surface at the point of entry, and therefore further downwards in the diagram, while at leaving it again it is refracted into a direction *further from* the perpendicular to the surface at the point of emersion, and therefore again more downwards in the diagram, so that the ray will follow some such course as *abc*D. In like manner the ray *a'b'*, which is incident below the axis of the lens, will, at entering and leaving the lens, be each time refracted according to the same law; the effect of which will be that its direction will become on each occasion more upwards or, as in the other ray, more towards the axis of the lens, following some such course as *a'b'c'*D. So that finally both rays will

tend to approach each other and meet in the point D. The rays between *ab* and *a'b'* will have their directions altered in precisely the same manner, but to a less and less extent in proportion as they are nearer the axis of the lens, the ray actually coinciding with that axis not being refracted at all; so that all the rays of the pencil will be made to converge to a point beyond the lens, which is called the focus. We have here supposed the rays to be parallel; if, however, they had been convergent, the same considerations will show that they would have been rendered more so, and if divergent, less so. When divergent rays proceed from a point and are refracted to a focus, that focus is said to be conjugate to the point or focus whence the rays first proceeded, since either point may be considered a focus with respect to the other, it being evident that any ray proceeding in any direction would, if it started in the opposite direction, pursue the very same course reversed. When parallel rays fall upon a convex lens as described above and become convergent, the focus so formed is called the *principal* focus of the lens. It is evident that all the above statements are merely slightly modified, and are essentially similar, when the axis of the pencil of rays is not accurately coincident with the axis of the lens.

9. Rays of light then proceeding from each of the various points of an object, and passing through a convex lens, will be collected into a focus on the other side of the lens, so that each point of the object will have its corresponding point or conjugate focus,

and thus a clear reproduction or image, as it is called, of the object will be formed.

10. We have thus early explained the action of a double convex lens, because we were anxious as soon as possible to make intelligible the way in which objects are actually made visible to our eyes.

11. At the back part of the eye the optic nerve ramifies out into a plexus called the retina, precisely as the ramifications of the olfactory nerves are spread over the inside of the nostrils. Sight is caused by the cognizance which these ramifications of the optic nerve take of the rays of light thrown upon them from different objects. But as single rays would give too feeble a light to make objects visible, and as moreover the rays from the different parts of an object would, if the eye consisted merely of this simple contrivance, fall upon and so interfere with each other that a confused sense of light alone would be perceived, the eye is also provided with more complex machinery for collecting the rays from definite points of objects and throwing them upon definite points of the retina, so as to produce there images or pictures of the things seen, which are then transmitted by the optic nerve itself to the brain. Our plan only admits of describing the most essential parts of this machinery. And the most important is a double convex lens called the *crystalline* lens, which is placed at such a distance from the retina, that rays parallel, or nearly so, falling upon it are collected into a focus on the retina. Thus very distant objects, the rays from any point of which

diverge so little as to be nearly parallel, are made visible. But as points from objects at a less distance send out rays of a degree of divergency the greater the smaller that distance is, the eye has the power, by altering either the place or the convexity of the crystalline lens, of so adapting itself as to bring to a focus rays of different amounts of divergency within a certain limit, thereby making such objects also visible. If the objects are too near (within, for most eyes, a distance of about five inches), the rays are so divergent that the limit is exceeded; then the rays from different points are thrown upon each other, and the vision is confused; moreover the effort instinctively made by the eye to adapt itself to a divergency exceeding its power becomes painful. No eye can bring to a focus rays possessing any, even the slightest *convergency*. To prevent the great and dazzling glare, which would take place if too much light were thrown upon the crystalline lens, there is placed before it an opaque substance like a curtain, termed the iris; in this is a circular aperture called the pupil, which the eye has the power of instinctively opening more or less (still preserving its roundness), so as to let in a greater or smaller number of rays, according to the amount of light to which it is exposed.

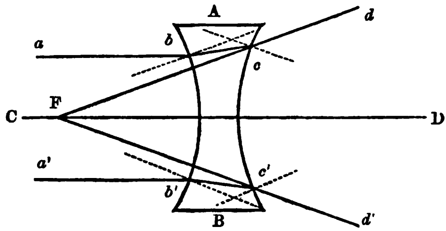
12. The degree of the convexity of the crystalline lens varies in different persons, which occasions defects of sight. For most eyes, all rays between the limits of parallelism and that amount of divergency which is produced by a distance of about five inches,

can be brought to a focus on the retina. But in some the lens is too convex, and rays which are parallel, or nearly so, are brought to a focus too soon, or before reaching the retina. This is called short sight, because such eyes can readily bring to a focus rays of greater divergency than that adapted to ordinary sight, and can therefore perceive distinctly objects nearer than the usual limit of distinct vision. In other eyes the lens is not sufficiently convex, and rays cannot be brought to a focus if they possess more than a slight amount of divergency; hence, by such eyes, objects at a distance are easily seen, but not those which are near. This, therefore, is called long sight; it is a defect usually brought on by old age.

13. The latter defect may be remedied by using spectacles the lenses of which are *convex* and therefore tend to make rays falling on them less divergent; the former by spectacles with *concave* lenses, the property of which is to increase the divergency of rays incident upon them.

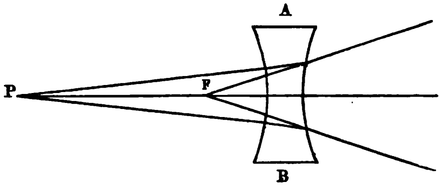
14. This effect of convex lenses has been already explained (§ 8), and it is equally easy to show what is the action of concave lenses. Let AB be a concave lens, and let a parallel pencil of rays the axis CD of which is coincident with the axis of the lens fall upon it. The ray CD is of course not refracted at all; the ray *ab*, which falls upon the point *b*, is refracted on entering the lens into a direction *bc* more nearly coincident with the perpendicular upon the tangent to the spherical surface at that point

(the perpendiculars to the tangents being indicated, as before, by the dotted lines); and on passing out of the lens on the other side, the ray follows a direction *cd* further from the perpendicular to the surface



at the point of emergence. The rays falling upon the other side of the axis of the lens are similarly affected; the ray *a'b'*, for instance, following the course *a'b'c'd'*; and the result is that the rays after passing through the lens become divergent, and move as if they had proceeded from the point F. This point then is called, as in the convex lens, the principal focus of the lens; but it is denominated a *virtual* focus, because the rays do not *actually* come from there, but move *as if* they did,—their direction, after traversing the lens, being precisely the same as if they really had diverged from that point. If the rays falling upon a concave lens had been convergent, they would have been rendered, by passing through it, less convergent, or parallel, or even divergent, according to the degree of concavity of the lens. If they had been divergent, they would have been rendered more so. Thus, if a conical pencil of divergent

rays proceed from the point P, and fall upon the concave lens AB, the rays will, after passing through it, move as if they had diverged from a point F nearer the lens, which is called the virtual focus conjugate

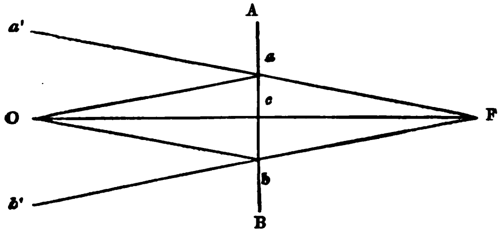


to P, since if the rays had proceeded in the reverse direction, and had converged before incidence upon the lens towards F, they would, after emerging from it, have converged towards P, so that the points P and F may be considered conjugate to each other.

15. Having now stated the laws of reflexion and refraction, and proceeded to describe the means by which the eye is enabled to obtain a clear perception of objects placed before it, we may now show more in detail how the images which the objects would themselves present to the eye are affected when the light by which they are seen has been reflected by a mirror or refracted through a lens. When the rays are brought by either of these means to a focus, a distinct image is then formed of the object, which is viewed by the eye as if it were itself the object. It becomes, then, our duty now to show, in the different cases of reflexion and refraction, what is the place, the magnitude, and the position of the image. The investigation of the effects of reflexion

is called Catoptrics; that of the effects of refraction Dioptrics.

16. We shall commence with reflexion, the simplest case of which is the reflexion produced by a plane mirror. Let AB be a plane mirror, and O a point in an object from which rays diverge in conical pencils upon the mirror. Let Oc be the ray which

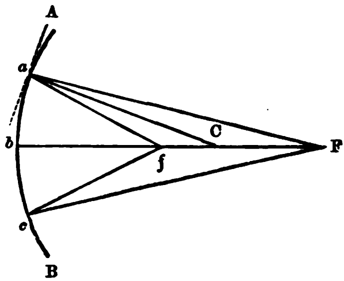


is perpendicular to the mirror, Oa, Ob rays of the same pencil at equal distances on each side of it. By the law of reflexion, Oc will be reflected back in the opposite direction, cO; Oa will be reflected in the direction aa', so that the angles OaB, $a'a$A are equal to each other; and Ob will be reflected in the direction bb', so that the angles ObA, $b'b$B are likewise equal. So that after reflexion the rays will move as if they proceeded from a point F on the other side of the mirror, which is therefore called the virtual focus; virtual, because they do not actually come from there, but move as if they did. The image of the object may therefore be said to be at F. Its place is evidently as far behind the mirror as the object itself is before it. And since all the points in the object have corresponding points in the

image, formed in precisely the same manner, the image of the whole object will be upright and of the same magnitude as the object itself.

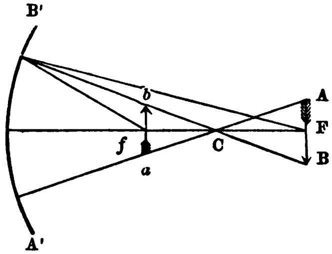
17. If the mirror be concave or convex, the formation of the image will be somewhat different: we will commence by describing it in the case of a concave mirror.

18. Let a pencil of rays diverge upon the concave mirror AB from the radiant point F. The ray F*b* which passes through C the centre of the spherical surface of the mirror is of course reflected back in the opposite direction, being perpendicular to the surface, or to the tangent at the point of incidence. But the ray F*a* makes an angle with the tangent at the point *a* smaller than a right angle by the angle C*a*F; it is therefore reflected back in the direction *af*, so that the angle F*a*C is equal to the angle *fa*C. It meets F*b* therefore in *f*; and it may in like manner be shown that the ray F*c* will be reflected so as to pass through the same point, which is consequently the focus corresponding to F, and the place where the image of the object is formed.



19. We will now consider how each individual point of the object is represented in the image. The

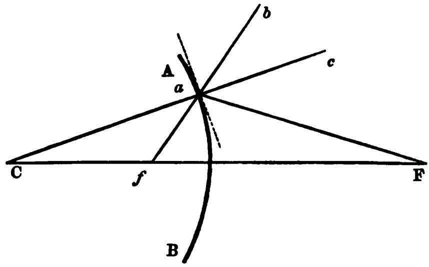
ray from A in the object AB which is reflected back in the opposite direction is evidently the one which passes through the centre C of the mirror; the same is the case with the ray from B in the other extremity of the object, and with those from all other points in it.



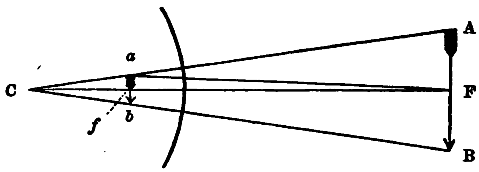
At f, therefore, where the image is formed, the rays proceeding from A will be brought to a focus in a, and those from B in b; ab therefore will represent the length of the image, and it is obvious that it is inverted and in the proportion to the object of fC to FC. Now, if B'f, B'F be joined, the angles fB'C, FB'C will, by the law of reflexion, be equal to each other; so that by Euclid vi. 3, fC : CF : : fB' : B'F. It is evident then that in this case the image is smaller than the object. F and f being conjugate foci, if the object were at f, the image would be at F; so that if the object be nearer the mirror than its spherical centre, the image will be longer than the object, but still inverted.

20. It remains that we speak of reflexion from a convex mirror. Let AB be such a mirror, and rays diverge upon it from the point F. If C be the centre of the mirror, Cac will be perpendicular to the tangent at a; and the ray Fa, incident at the point a, will be reflected into the direction ab, so that the

angles F*ac*, *cab* will be equal to each other; *ba* then being produced to meet CF in *f*, the image will be



formed in *f*, which is therefore a virtual focus conjugate to F. The image is therefore on the other side of the mirror and nearer to it than the centre of the mirror. Now let AB be an object the image of which, after reflexion at a convex mirror, whose centre is C, is formed at the distance F*f*. The rays



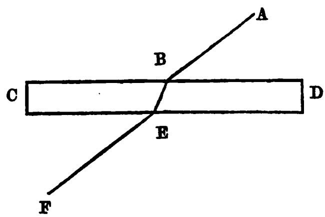
from A, B, which are reflected in the opposite direction to that in which they were incident, are those which proceed towards the centre of the mirror, so that *ab* will represent the image. The description shows that it is erect, and smaller than the object itself in the proportion of C*f* to CF.

21. Before quitting the subject of reflexion, it should be mentioned that the point where parallel

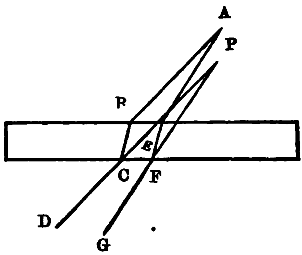
rays are brought to a focus is called the *principal* focus of the mirror.

22. Having traced the formation of the image in the different cases of reflexion, we proceed to do the same in some of the cases of refraction.

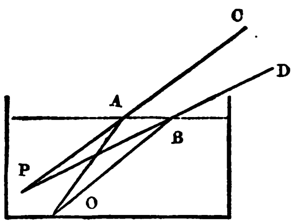
23. And first of refraction through a medium bounded by a plane surface. Let the ray of light AB fall upon the refracting medium CD, which is bounded by two parallel plane surfaces. By the law of refraction it will, after entering the medium, follow the course BE, being refracted into a direction *less* inclined than before to the perpendicular to the surface. After passing through the medium it will be refracted into a direction *more* inclined to that perpendicular, and will in fact move along EF, parallel to its original course before entering the medium. It is evident therefore that if rays of light diverge from a radiant point and pass through a medium bounded by two parallel plane refracting surfaces, each ray will move in the same direction after as before passing through the medium, and all will emerge neither more nor less divergent than they entered it. An object thus seen will, however, appear displaced; for the ray AB (see fig. next page) moving after refraction in the direction CD parallel to AB, and the ray AE in the direction FG parallel to AE, if DC, GF be pro-



duced till they intersect in P, the rays from A will appear to proceed from P, and the same will be the case with every point in the object, which will therefore be seen as if displaced in the direction AP. P may be considered analogically as a virtual focus conjugate to A. But the apparent distance and magnitude of the object will be but slightly changed.



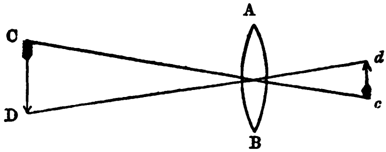
24. Another circumstance is deserving of attention here. Let an object immersed in a fluid or other refracting substance be viewed from a point outside, and let O represent any point in that object. The rays OA, OB will, on leaving the surface, be refracted into a direction less inclined to the surface (more to the perpendicular to that surface), but OB to a greater extent than OA; they will consequently after emergence pursue the courses AC, BD respectively, having become more divergent than before. They will therefore appear to proceed from a point P nearer the surface of the fluid. This accounts for the fact that, when an object is seen below the surface of a



fluid, it looks as if it were considerably higher than it really is.

25. It is unnecessary here to discuss in detail any more cases of refraction through media bounded by plane surfaces, which may all be treated in the same manner as the above case of the two surfaces being parallel to each other. But we must say something more concerning refraction through lenses. It has been already shown how the rays of light are affected by passing through double convex and double concave lenses, but without investigating the magnitude and position of the images thus formed: this we now proceed to do.

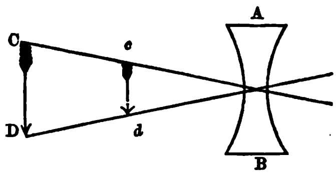
26. In § 8, we showed that when rays of light diverge from a radiant point and pass through a double convex lens, they will emerge from it less divergent than they entered it. If they come from a point at the same distance from the lens as its principal focus, it results from the principles laid down in that article, that they will emerge parallel; if from a greater distance, convergent. In the latter case, then, an image will be formed on the other side of the lens, the position and magnitude of which is



thus investigated. Let AB be a double convex lens, and CD an object from which rays of light, after

passing through it, form an image at *cd*. Since the rays which are not refracted are those, C*c*, D*d*, which pass through the centre of the lens (and the rays which traverse other parts of the lens are so refracted as to meet these in a focus at the place where the image is formed), it is clear that the image will be in an inverted position with respect to the object. Its real relative magnitude will depend upon the proportional distance of the image and the object from the lens. If the object is at a great distance from the lens, the image will be nearer and will be smaller than the object; but if the object is at a moderate distance from the lens, the image will be further from it and will be larger than the object. The *apparent* magnitude of the image will be increased by its greater proximity to the eye than the object.

27. The case of the double concave lens has also been discussed before (see § 14). It remains that we in this case likewise investigate the position and magnitude of the image formed. Let A B be a double concave lens, and let CD be an object viewed through it.



The rays which are not refracted at all are those, C*c*, D*d*, which pass through the centre of the lens, and it was shown in § 14 that the image is formed at *cd* between the object and the lens. It is evident therefore that it will be erect, and that its real magnitude will be smaller than that of the object

itself. The eye being on the other side of the lens, the apparent magnitude will be still more diminished.

28. We have shown (§ 26) that the image formed by a convex lens is inverted as compared with the object; and in describing the eye (§ 11) that vision is performed by the images of the objects seen being thrown upon the retina or expansion of the optic nerve. Many persons therefore have been puzzled to understand why we do not see objects in an inverted position instead of erect. But too little is known of the way in which the image, after being painted on the retina, is ultimately conveyed to the brain so as to produce the sense of sight, to enable us to account for this; at the same time the known fact of our ignorance of that process may well suffice to remove any perplexity that might be felt from this circumstance.

29. When the image formed by a convex lens is at a greater distance from the lens than the object is which produces it, it is larger than the object, and thus a more or less magnified image is formed. A well-known illustration of this is the magic lantern.

30. But there is another way in which magnification may be produced by a convex lens, of which advantage is taken in the construction of spectacles and other instruments to assist the sight. Allusion to this has already been made in § 12, speaking of defects of sight, but it is necessary clearly to distinguish between this and the formation of a magnified image of the object. Long sight or short sight may

be remedied, as we have said, by the use of convex or concave glasses, which so alter the divergency of the rays of light proceeding from the various points of an object seen, that the eye can adapt itself to them and bring them to a focus on the retina, where an image of the object is formed. The eye thus sees objects in other respects as it would if unassisted—in their proper position, and of the apparent magnitude due to the distance at which they are placed. Now if a lens of great convexity is placed near an object, rays of great divergency may have their divergency so much diminished that the eye will be able to bring them to a focus, and afford a distinct view of the object. If the object be at the distance of the principal focus of the lens from it, the rays will, in accordance with the principles previously laid down, emerge parallel; if a little nearer, slightly divergent. By so placing, then, an object a little nearer than the principal focus of a lens of great convexity and therefore of very small focal distance, the object may be viewed through it, and the rays emerging but slightly divergent, the eye may be brought close to the lens on the other side, and thus an image be formed *on the retina* magnified, because the object is nearer than the least distance of natural distinct vision. This is the principle of the pocket lens.

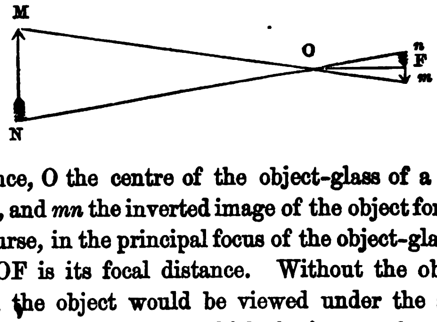
31. When an image has been formed of an object by the use of a convex lens, there is nothing to prevent that image being magnified by means of another convex lens of small focal distance in the way last described: and this is in fact done in the microscope

and telescope. Both these instruments consist essentially of two convex lenses; one nearest the object, called the object-glass, by which the image is formed; the other nearest the eye, called the eye-glass, by which that image is magnified. The great difference is, that in the microscope the two glasses are so arranged that rays of some degree of divergency are brought to a focus by the object-glass, and the eye-glass is placed so that the image thus formed is very nearly in its principal focus, thus transmitting the rays to the eye nearly in a state of parallelism, and fitted to be brought again to a focus on the retina; whilst in the telescope the object-glass having formed in its principal focus an image of a very distant object, the rays proceeding from which are almost parallel, that image is magnified by the eye-glass in the same manner as in the microscope. Since the object-glass forms an inverted image, and this is not changed by the eye-glass, the objects are seen in an inverted position. Our plan does not admit of explaining more than the naked principle of the microscope and telescope; matters of detail must be sought elsewhere. It may, however, be remarked that by the use of another pair of lenses in the microscope, the object may be restored to its natural position: the telescope being chiefly used for astronomical purposes, the inversion of the image is of little importance. The telescope we have thus briefly described is called the *refracting*, or sometimes, from its ordinary use in astronomy, the *astronomical* telescope. In the *reflecting* telescope the image is formed by means of a

concave mirror, and then magnified as before by an eye-glass.

32. Since the eye-glass magnifies only because it enables us to bring the object nearer the eye, the amount of its magnification is the proportion of the least distance of distinct vision (for ordinary eyes about 5 inches) to the distance at which the object is placed when seen through the lens, or very nearly to the focal distance of the eye-glass.

33. It may be readily shown in what proportion the image of a distant object formed by an object-glass of a telescope appears larger than the object itself, when that image is viewed without the aid of an eye-glass. Let MN be an object at a very great



distance, O the centre of the object-glass of a telescope, and *mn* the inverted image of the object formed, of course, in the principal focus of the object-glass, so that OF is its focal distance. Without the object-glass, the object would be viewed under the angle MON$=$*m*O*n*$=$the angle which the image subtends at the centre of the object-glass. It appears therefore to the unaided eye of the same magnitude as the image would if seen from the distance OF. But the image may be viewed by the eye on the other side of F, at the least distance of distinct vision. So that

the object will appear greater than it naturally would in the proportion of the focal distance of the object-glass to the least distance of distinct vision.

34. In what precedes, where we have spoken of magnification, linear magnification only was meant. If the length of an object be magnified a certain number of times, the surface of that object will be magnified the square of that number of times.

35. We will now in conclusion devote a very few words to the *dispersion* of light, by which the formation of colours, called chromatism, takes place. Hitherto we have spoken of light as if there was but one kind of light; but it is well known that there are various kinds of coloured light. Common white light consists of all these kinds blended; but as the different kinds possess different degrees of refrangibility, if a beam of light is passed through a prism or other refracting body, it is separated into its component parts, producing the phenomenon of colours, the different coloured rays being arranged according to their respective amounts of refrangibility: this is called the dispersion of light. The order of the colours is well known. Commencing with the least refrangible, it is red, orange, yellow, green, blue, indigo, and violet. A pencil of light being passed through a prism and then received upon a screen, an elongated stripe of colours, in the above order, will be seen; this is termed the prismatic spectrum.

36. It may be well to give some idea of the amount of the variation of refrangibility of the dif-

ferent coloured rays. We have already defined (§ 5) the refractive index of a substance; for water it is about 1·3, and for glass about 1·5. Now in passing through crown-glass, the refractive index of the red rays is 1·54, and that of the violet rays 1·56.

THE END.

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